## Math 53 Discussion: Review for final

1) Find the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

2) Maximize $f(x, y, z) = xyz$ subject to $x^2 + y^2 + z^2 = 3$.

3) Evaluate $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1}\, dy\, dx$ by reversing the order of integration.

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4) Find the work done by the force field $\overrightarrow{F}(x,y) = x\,\hat{\mathbf{i}} + (y+2)\,\hat{\mathbf{j}}$ in moving an object along an arch of the cycloid $\overrightarrow{r}(t) = (t - \sin t)\,\hat{\mathbf{i}} + (1 - \cos t)\,\hat{\mathbf{j}}$, $0 \le t \le 2\pi$.

5) Use Stokes' Theorem to evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ where $\overrightarrow{F}(x,y,z) = xy\,\hat{\mathbf{i}} + yz\,\hat{\mathbf{j}} + zx\,\hat{\mathbf{k}}$ and $C$ is the triangle with vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ oriented counterclockwise as viewed from above.

6) Suppose we're given a function $f(x,y,z)$, vector field $\overrightarrow{F}(x,y,z)$ and curve $C$ given by $\overrightarrow{r}(t) = \langle x(t), y(t), z(t) \rangle$ for $a \le t \le b$. a) What is the line integral of $f$ along $C$? b) What is the work done by $\overrightarrow{F}$ along $C$?

**Answers:** 1) $\pm(2, 1/2, -1)$. 2) max value is 1, min value is $-1$. 3) $\frac{1}{3}\ln 9$. 4) $2\pi^2$. 5) $-1/2$. 6) $\int_a^b f(x(t),y(t),z(t))\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$, $\int_a^b \overrightarrow{F} \cdot \overrightarrow{r}'(t)\,dt$.