Math 53 Discussion

Practice Problems: 16.8, Stokes' theorem

1) [*In section.*] Let S be the portion of the paraboloid $z = 4 - x^2 - y^2$ above the plane $z = 0$, with upward normal. Let $\overrightarrow{F} = \langle y - z, -(x + z), x + y \rangle$. Compute $\int\int_S \operatorname{curl} \overrightarrow{F} \cdot d\overrightarrow{S}$.

2) [*In section.*] Find $\int\int_S \operatorname{curl} \overrightarrow{F} \cdot d\overrightarrow{S}$ where $\overrightarrow{F} = (xyz)\,\hat{\mathbf{i}} + (xy)\,\hat{\mathbf{j}} + (x^2yz)\,\hat{\mathbf{k}}$ and S consists of the top and 4 sides (no bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$ oriented outward.

Over →

3) [*In section.*] Let C be the triangle in $\mathbb{R}^3$ with vertices $(1,0,0)$, $(0,2,0)$ and $(0,0,1)$. Compute

$$\int_C (x^2+y)\,dx + yz\,dy + (x-z^2)\,dz$$

4) Use Stokes' Theorem to evaluate $\int_C \overrightarrow{F}\cdot d\overrightarrow{r}$ where $\overrightarrow{F}(x,y,z) = x^2z\,\hat{\mathbf{i}} + xy^2\,\hat{\mathbf{j}} + z^2\,\hat{\mathbf{k}}$ and C is the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=9$ oriented counterclockwise as viewed from above.

Answers: 1) -8π. 2) 0. 3) $-13/6$. 4) $81\pi/2$.