Math 53 Discussion: Review

Quiz on Wednesday: Review 16.6, 16.7, 16.9

1a) Find the centroid of a right circular cone with height h and base radius a. (Place the cone so that its base is in the xy-plane with center the origin and its axis along the positive z-axis.) b) Find the moment of inertia of the cone about its axis (z-axis).

2) Use polar coordinates to evaluate $\displaystyle\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2)\, dy\, dx$.

3) Rewrite the integral $\displaystyle\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)\, dz\, dy\, dx$ as an iterated integral in order $dx\, dy\, dz$.

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4) Evaluate $\int_C xy\,dx + y^2\,dy + yz\,dz$ where C is the line segment from $(1, 0, -1)$ to $(3, 4, 2)$.

5) Show that $\overrightarrow{F}(x, y, z) = \sin y\,\hat{\mathbf{i}} + x\cos y\,\hat{\mathbf{j}} - \sin z\,\hat{\mathbf{k}}$ is conservative, and find a potential function f so that $\overrightarrow{F} = \nabla f$.

6) If f is a harmonic function, that is, $\nabla^2 f = 0$, show that the line integral $\int_C f_y\,dx - f_x\,dy$ is independent of path C in any simple region D.

7) Evaluate $\int\int_S \overrightarrow{F} \cdot d\overrightarrow{S}$ where $\overrightarrow{F} = xz\,\hat{\mathbf{i}} - 2y\,\hat{\mathbf{j}} + 3x\,\hat{\mathbf{k}}$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.

Answers: 1a) $(0, 0, h/4)$, b) $\pi a^4 h/10$. 2) $486/5$. 3) $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}}$. 4) $110/3$. 5) $x\sin y + \cos z + k$. 6) By Green's theorem, if C is a simple closed curve then the line integral is zero. 7) $-64\pi/3$.