Math 53 Discussion: Review

1) Use the transformation $u = x - y$, $v = x + y$ to evaluate $\int\int_R \frac{x-y}{x+y} dA$ where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$ and $(1, 3)$.

2) Find $\int\int\int_E yz\, dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.

3) Describe the following surfaces in spherical coordinates: (i) the plane $z = a$, (ii) the sphere of radius a centered at $(0, 0, a)$, (iii) the cone $z = \sqrt{x^2 + y^2}$.

4) Set up $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ where $\overrightarrow{F}(x, y, z) = e^z\, \hat{\mathbf{i}} + xz\, \hat{\mathbf{j}} + (x + y)\, \hat{\mathbf{k}}$ along curve C given by $\overrightarrow{r}(t) = t^2\, \hat{\mathbf{i}} + t^3\, \hat{\mathbf{j}} - t\, \hat{\mathbf{k}}$ for $0 \leq t \leq 1$.

5) Use the Divergence theorem to calculate the flux of $\overrightarrow{F} = e^x \sin y\ \hat{\mathbf{i}} + e^x \cos y\ \hat{\mathbf{j}} + yz^2\ \hat{\mathbf{k}}$ across the surface $S =$ box bounded by the planes $x = 0, x = 1,\ y = 0, y = 1$ and $z = 0, z = 2$.

6) Find the area of the surface given by the part of the plane $x + 2y + 3z = 1$ that lies inside the cylinder $x^2 + y^2 = 3$.

7) Find the flux of $\overrightarrow{F} = \langle ze^{xy}, -3ze^{xy}, xy \rangle$ through the parallelogram with parametric equations $x = u + v, y = u - v$ and $z = 1 + 2u + v$ for $0 \le u \le 2$ and $0 \le v \le 1$, with upward orientation.

Answers: 1) The four sides are $y = x, y = 4 - x, y = 2 + x, y = 2 - x$. These map to the square $-2 \le u \le 0$ and $2 \le v \le 4$ in the uv-plane. Get $\int_2^4 \int_{-2}^0 \frac{u}{v} \frac{1}{2} dudv = -\ln 2$. 2) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^y yz\ dzdydx$. Do z-integral, then convert to polars to get $\int_0^\pi \int_0^2 \frac{1}{2} r^3 (\sin^3 \theta) r dr\ d\theta$. Use $\sin^3 \theta = \sin \theta (1 - \cos^2 \theta)$ to get $\frac{64}{15}$. 3) (i) $\rho = a \sec \phi,\ \rho = 2a \cos \phi,\ \phi = \pi/4$. 4) $\int_0^1 2te^{-t} - 3t^5 - t^2 - t^3\ dt$. 5) Since div $\overrightarrow{F} = 2yz$, we get $\int_0^2 \int_0^1 \int_0^1 2yz = 2$. 6) Graph of $z = \frac{1}{3}(1 - x - 2y)$ so $dS = \sqrt{1 + 1/9 + 4/9}\ dA$ and $\int\int_{x^2+y^2 \le 3} \sqrt{14}/3\ dA = \pi(\sqrt{3})^2 \sqrt{14}/3 = \pi\sqrt{14}$. 7) Parametrize $\overrightarrow{r}(u, v) = (x(u, v), y(u, v), z(u, v))$. $\overrightarrow{r}_u = \langle 1, 1, 2 \rangle$, $\overrightarrow{r}_v = \langle 1, -1, 1 \rangle$, so $d\overrightarrow{S} = \pm \langle 3, 1, -2 \rangle\ dudv$. Take $-$ so that z component is > 0, since we want upward orientation. Get $\int_0^1 \int_0^2 2(u^2 - v^2) dudv = 4$.