Math 53 Discussion

Practice Problems: 16.7, 16.9, surface integrals, flux, divergence theorem

1) [*Will do in section.*] Evaluate the surface integral $\int\int_S y \, dS$ where S is the part of the paraboloid $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 4$.

2) [*Will do in section.*] Evaluate the flux of $\overrightarrow{F}(x, y, z) = xze^y \, \hat{\mathbf{i}} - xze^y \, \hat{\mathbf{j}} + z \, \hat{\mathbf{k}}$ across the surface S consisting of the part of the plane $x + y + z = 1$ in the first octant and with downward orientation.

Over →

3) [*Will do in section.*] Use the Divergence theorem to find the flux of $\overrightarrow{F} = (e^z + y^2 x)\ \hat{\mathbf{i}} + (\cos x + x^2 z)\ \hat{\mathbf{k}}$ through the surface S bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 1$.

4) Use the Divergence theorem to calculate the flux of $\overrightarrow{F} = |\overrightarrow{r}|\overrightarrow{r}$ through the surface S given by the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and the disk $x^2 + y^2 \leq 1$ in the xy-plane. (Here $\overrightarrow{r}$ denotes the vector $\langle x, y, z \rangle$).

Answers: 1) $\frac{\pi}{16}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^{17} = \frac{\pi}{60}(391\sqrt{17} + 1)$. 2) $-1/6$. 3) $\frac{\pi}{10}$. 4) 2π.