Math 53 Discussion

Quiz on Monday: Review 16.1–16.5

Practice Problems: 16.5, curl and divergence

1)[*From Wednesday's handout.*] If $\overrightarrow{F}(x,y) = \dfrac{-y\hat{\imath} + x\hat{\jmath}}{x^2+y^2}$ (c.f. HW 10), show $\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = 2\pi$ for <u>all</u> positively-oriented simple closed paths about the origin.

2i) Find the curl of $\overrightarrow{F}(x,y) = \sin(xy)\hat{\imath} + \cos(xy)\hat{\jmath}$.

ii) Find the curl of $\overrightarrow{F}(x,y,z) = xy\,\hat{\imath} + x^2z\,\hat{\jmath} - (y+z^3)\,\hat{k}$.

3) Calculate the divergence of the vector field obtained in 2ii).

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4) Compute the flux of $\overrightarrow{F}(x,y) = x\,\hat{\imath} + y\,\hat{\jmath}$ through 4 sides of the unit square, one vertex at $(0,0)$. Do it directly and then using Green's theorem.

5) Let R be the region enclosed by the x-axis, $x = 2$, and $y = \sqrt{x}$.

(a) Use Green's theorem to compute the flux $\oint \overrightarrow{F} \cdot \hat{n} ds$ of $\overrightarrow{F} = xy\,\hat{\imath}$ out of R.

(b) Find the flux of $\overrightarrow{F}$ out of R through the two segments C_1 (horizontal) and C_2 (vertical).

(c) Using (a) and (b), find the flux out of the third side C_3.

Answers: 1) Use Green's theorem applied to a region with two boundaries, one is the arbitrary curve C and the other is a small circle inside curve. 2i) $-y\sin(xy) - x\cos(xy)$, ii) $-(1+x^2)\hat{\imath} + (-x + 2xz)\hat{k}$. 3) 0. 4) 2. 5a) 1, b) 0 and 2, c) -1