Math 53 Discussion

Practice Problems: 16.4, Green's theorem

1) Evaluate the following two integrals (a) directly and (b) using Green's theorem.

(i)[*Will do in section.*] $\int_C xy^2\,dx + 2x^2y\,dy$ where C is the triangle with vertices $(0,0)$, $(2,2)$ and $(2,4)$.

(ii) $\oint_C (x-y)\,dx + (x+y)\,dy$ where C is the circle centered at the origin of radius 2.

Over →

2) Use Green's theorem to evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ where $\overrightarrow{F}(x,y) = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle$ and C is the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$.

3) If $\overrightarrow{F}(x,y) = \dfrac{-y\hat{\imath} + x\hat{\jmath}}{x^2 + y^2}$ (c.f. HW 10), show $\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = 2\pi$ for <u>all</u> positively-oriented simple closed paths about the origin.

Extra Green's theorem practice (won't go over in class, but answers below.)

4) Use Green's theorem to find the work done by the force $\overrightarrow{F}(x,y) = x(x+y)\hat{\imath} + xy^2\hat{\jmath}$ in moving a particle from the origin along the x-axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin along the y-axis.

Answers: 1) i) 12, ii) 8π. 2) Use Green's thm to evaluate integral around a closed curve, then $\int_C = \int_{closed\ curve} - \int_{line\ segment}$. Answer: $\pi/2 + e^{\pi/2} - e^{-\pi/2}$. 3) Use Green's theorem applied to a region with two boundaries, one is the arbitrary curve C and the other is a small circle inside curve. 4) $-1/12$.