Math 53 Discussion

No quiz next week (Veteran's day), but HW due Weds

Practice Problems: 16.3, fundamental theorem of line integrals, conservative vector fields

1) Determine if $\overrightarrow{F}$ is conservative.

(a) $\overrightarrow{F}(x, y) = e^x \cos y\hat{i} + e^x \sin y\hat{j}$.

(b) $\overrightarrow{F}(x, y) = (\ln y + 2xy^3)\hat{i} + (3x^2y^2 + x/y)\hat{j}$.

2) For the conservative $\overrightarrow{F}$ above, find its scalar potential function f, i.e. $\overrightarrow{F} = \nabla f$.

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3) (a) Find a function f such that $\overrightarrow{F} = \nabla f$, where $\overrightarrow{F}(x,y) = xy^2\hat{i} + x^2y\hat{j}$.

(b) Evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ along the curve $C : \overrightarrow{r}(t) = \langle t + \sin \pi t/2, t + \cos \pi t/2 \rangle$ for $0 \le t \le 1$.

4) Explain why the following holds: Suppose a vector field $\overrightarrow{F}(x,y)$ is perpendicular to the tangent vector $\overrightarrow{r}'(t)$ to a curve C, at each point $(x(t), y(t))$ on the curve. Then $\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = 0$.

Extra line integral practice (won't go over in class, but answers below.)

5) Evaluate $\int_C (xy + \ln x)dy$ where C is the arc of the parabola $y = x^2$ from $(1,1)$ to $(3,9)$.

6) The position of an object with mass m at time t is $\overrightarrow{r}(t) = at^2\hat{i} + bt^3\hat{j}$, for $0 \le t \le 1$. (a) Find the force acting on the object at time t. (b) What is the work done by the force during $0 \le t \le 1$?.

Answers: 1) No, Yes. 2) $x \ln y + x^2y^3 + C$. 3) a) $x^2y^2/2$, b) 2. 4) dot product in integral is zero along C. 5) $464/5 + 9 \ln 3$. 6) a) Use $\overrightarrow{F} = m\overrightarrow{a}$ to get $2ma\hat{i} + 6bmt\hat{j}$, b) $m(2a^2 + \frac{9}{2}b^2)$.