

Math 53 Discussion

Practice Problems: 16.1–16.2, vector fields, line integrals

1) Sketch the vector field $\vec{F}(x, y) = x^2\hat{i}$.

2) Sketch the contour plot and gradient vector field of $f(x, y) = xy$. For example, you could start by sketching the contours $f(x, y) = 1$ and $f(x, y) = 2$.

3) Find $\int_C z \, ds$ where C is the helix $(\cos t, \sin t, t)$ for $0 \leq t \leq \pi$.

4) Find $\int_C xy \, dx + (x - y) \, dy$, where $C =$ line segments from $(0, 0)$ to $(2, 0)$ and $(2, 0)$ to $(3, 2)$.

5) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2 y^3 \hat{i} - y\sqrt{x} \hat{j}$ and $\vec{r}(t) = t^2 \hat{i} - t^3 \hat{j}$ for $0 \leq t \leq 1$.

1) arrows point right only, increasing in length with $|x|$. 2) vector field arrows perpendicular to contour lines. 3) $\frac{\pi^2 \sqrt{2}}{2}$. 4) $\frac{17}{3}$. 5) $-\frac{59}{105}$.