Math 53 Discussion

Practice Problems: 15.7–15.8, applications of triple integrals, cylindrical coordinates

1) Find the mass and set up the x-coordinate of the center of mass of the solid E with constant density function $\rho = 2$, where:

E lies under the plane $1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 1$.

2) a) Find the volume of the region E bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

b) Set up the integrals to find the centroid of E (the center of mass in the case where density is constant.)

3) Find the moments of inertia of a cube of constant density k and side length L, if one vertex is located at the origin and three edges lie along the coordinate axes.

1) $m = 79/30$, $\bar{x} = \frac{1}{m} \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 2x \, dz \, dy \, dx$

2) 162π, $\bar{x}, \bar{y}$ should be 0 by symmetry. $\bar{z} = \frac{1}{162\pi} \int_0^{2\pi} \int_0^3 \int_{r^2}^{36-3r^2} zr \, dz \, dr \, d\theta$.

3) $I_x = I_y = I_z = \frac{2k}{3} L^5$