

Math 53 Discussion

Practice Problems: 15.5, applications of double integrals

1) Find the mass of the lamina that occupies $D =$ triangular region with vertices $(0, 0)$, $(2, 1)$ $(0, 3)$ and has density function $\rho(x, y) = x + y$.

2) Find the center of mass of the lamina with density function $\rho(x, y) = ky$ that occupies the region D which is bounded by $y = 1 - x^2$ and $y = 0$.

3) Find the center of mass of a lamina in the shape of an isosceles right triangle with equal sides of length a , if the density at any point is proportional to the square of the distance from the vertex opposite the hypotenuse. [*Use symmetry - you only need to find one coordinate in the center of mass.*]

4) Find the moments of inertia I_x, I_y, I_0 for a lamina of constant density ρ occupying the region in the first quadrant with $x^2 + y^2 \leq a^2$.

5) The joint density function for a pair of random variables X and Y is given by:

$$f(x, y) = \begin{cases} Cx(1 + y) & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of the constant C .
- (b) Find $P(X \leq 1, Y \leq 1)$.
- (c) Find $P(X + Y \leq 1)$.