Math 53 Discussion: Review problems for Midterm 1

1) Find the area of the the region lying inside both $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$.

2) Find the length of the curve $x(t) = 3t^2$, $y(t) = 2t^3$ for $0 \le t \le 2$.

3) a) Find a vector perpendicular to the plane through the points $A(1, 0, 0)$, $B(2, 0, -1)$ and $C(1, 4, 3)$. b) Find the area of the triangle ABC.

4) Find an equation of the plane through $(3, -1, 1)$, $(4, 0, 2)$ and $(6, 3, 1)$.

5) A surface consists of all points P such that the distance from P to the plane $y = 1$ is twice the distance from P to the point $(0, -1, 0)$. Find the equation for the surface and identify.

6) Find the point where the line $\overrightarrow{r}(t) = \langle 2 - t, 1 + 3t, 4t\rangle$ intersects the plane $2x - y + z = 2$.

7) A particle moves with position function $\overrightarrow{r}(t) = t\ln t\ \hat{\mathbf{i}} + t\ \hat{\mathbf{j}} + e^{-t}\ \hat{\mathbf{k}}$. Find the velocity, speed and acceleration of the particle.

8) An athlete throws a shot at an angle of 45° to the horizontal at an initial speed of 43 ft/s. The ball leaves his hand 7 ft above the ground. Assuming acceleration comes from gravity only, find the position vector describing the ball's trajectory.

9) Question 49 from Chapter 14 review of the textbook (reading a contour plot of hurricane wind speed.)

10) Use polar coordinates to find the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2}$.

11) Find the equation of the normal line and tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at $(1, -2, 1)$.

12) Find the linear approximation of $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at $(2, 3, 4)$.

13) Use the Chain Rule to find $\frac{du}{dp}$ where $u(x, y) = x^2y^3, x(p) = p + 3p^2, y(p) = pe^p$.

14) Find the directional derivative of $f(x, y) = x^2e^{-y}$ in the direction towards $(2, -3)$ from the point $(-2, 0)$.

15) Find the absolute maximum and minima of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on D, the closed triangular region with vertices at $(0, 0), (0, 6)$ and $(6, 0)$.

Answers: 1) $\frac{1}{2}(\pi-1)$, 2) $2(5\sqrt{5}-1)$, 3) $\langle 4,-3,4\rangle$, $\frac{\sqrt{41}}{2}$, 4) $-4x+3y+z=-14$, 5) $4x^2+3y^2+10y+4z^2+3=0$, ellipsoid. 6) $(1,4,4)$. 7) $\overrightarrow{r}'(t)=(1+\ln t)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}-e^{-t}\,\hat{\mathbf{k}}$, $|\overrightarrow{v}(t)|=\sqrt{(1+\ln t)^2+1+e^{-2t}}$, $\overrightarrow{a}(t)=\frac{1}{t}\,\hat{\mathbf{i}}+e^{-t}\,\hat{\mathbf{k}}$, 8) $\overrightarrow{r}(t)=\frac{43}{\sqrt{2}}t\,\hat{\mathbf{i}}+\left(\frac{43}{\sqrt{2}}t-\frac{1}{2}gt^2+7\right)\hat{\mathbf{j}}$, 9) $\approx\frac{5}{8}$, 10) -1. 11) plane: $z=8x+4y+1$, normal line: $x=1+8t, y=-2+4t, z=1-t$, 12) $f(x,y,z)\approx 60x+(24/5)y+(32/5)z-120$, 13) $2xy^3(1+6p)+3x^2y^2(e^p+pe^p)$, 14) $-\frac{4}{5}$, 15) max at $(1,2)$ and min at $(2,4)$