Math 53 Discussion

1) Evaluate the following two integrals (a) directly and (b) using Green's theorem.

(i)[#5, §16.4] $\int_C xy^2\,dx + 2x^2y\,dy$ where C is the triangle with vertices $(0,0)$, $(2,2)$ and $(2,4)$.

(ii) $\oint_C (x-y)\,dx + (x+y)\,dy$ where C is the circle centered at the origin of radius 2.

2) [Similar to #12, §16.4] Use Green's theorem to evaluate $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$ where $\overrightarrow{F}(x,y) = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle$ and C is the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$.

3) [#17, §16.4] Use Green's theorem to find the work done by the force

$$\overrightarrow{F}(x,y) = x(x+y)\ \hat{\mathbf{i}} + xy^2\ \hat{\mathbf{j}}$$

in moving a particle from the origin along the x-axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin along the y-axis.

Answers: 1) i) 12, ii) 8π. 2) Use Green's thm to evaluate integral around a closed curve, then $\int_C = \int_{closed\ curve} - \int_{line\ segment}$. Answer: $\pi/2 + e^{\pi/2} - e^{-\pi/2}$. 3) $-1/12$.