Math 53 Discussion

Practice Problems: Sections 14.3-14.5. Equation of the tangent plane, linear approximations, Clairaut’s Theorem, the Chain Rule

1) [§ 33, §14.3] Find the first partial derivatives of $w = \ln(x + 2y + 3z)$.

2) Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ where $x^2 + y^2 + z^2 = 3xyz$.

3) [§ 5 §14.4] Find an equation of the tangent plane to the surface

\[ f(x, y) = z = x \sin(x + y) \]

at the point $(-1, 1, 0)$. 
4) Clairaut’s Theorem says that if \( f_{xy} \) and \( f_{yx} \) are continuous on some disk containing a point, then they are equal at that point. Verify Clairaut’s theorem holds with \( f(x, y) = e^x \cos(xy) \), for all \((x, y)\).

5) Find the linearization of \( f(x, y) = e^x \cos(xy) \) at \((0, 0)\).

6) Find the linear approximation of \( f(x, y, z) = x^3\sqrt{y^2 + z^2} \) at \((2, 3, 4)\) and use it to approximate \( f \) at \((1.98, 3.01, 3.97)\).

7) \([\#3, \S14.5] \) Find \( \frac{dz}{dt} \) where \( z = \sqrt{1 + x^2 + y^2} \) and \( x = \ln t, y = \cos t \).

Answers: 1) \( w_x = 1/(x + 2y + 3z), w_y = 2/(x + 2y + 3z), w_z = 3/(x + 2y + 3z) \). 2) \( z_x = (3yz - 2x)/(2z - 3yx), z_y = (3xz - 2y)/(2z - 3yx) \). 3) \( x + y + z = 0 \). 5) \( z = x + 1 \). 6) \( f(x, y, z) \approx 60x + \frac{24}{5}y + \frac{32}{5}z - 120 \). Plugging in \((1.98, 3.01, 3.97)\) for \((x, y, z)\) gives approximately 38.656. 7) \( [(x/t) - y \sin t]/\sqrt{1 + x^2 + y^2} \).