Math 53 Discussion

Practice Problems: Sections 14.3-14.5. Equation of the tangent plane, linear approximations, Clairaut's Theorem, the Chain Rule

1) [# 33, §14.3] Find the first partial derivatives of $w = \ln(x + 2y + 3z)$.

2) Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ where $x^2 + y^2 + z^2 = 3xyz$.

3) [# 5 §14.4] Find an equation of the tangent plane to the surface

$$f(x, y) = z = x \sin(x + y)$$

at the point $(-1, 1, 0)$.

4) Clairaut's Theorem says that if f_{xy} and f_{yx} are continuous on some disk containing a point, then they are equal at that point. Verify Clairaut's theorem holds with $f(x, y) = e^x \cos(xy)$, for all (x, y).

5) Find the linearization of $f(x, y) = e^x \cos(xy)$ at $(0, 0)$.

6) Find the linear approximation of $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at $(2, 3, 4)$ and use it to approximate f at $(1.98, 3.01, 3.97)$.

7) [#3, §14.5] Find $\dfrac{dz}{dt}$ where $z = \sqrt{1 + x^2 + y^2}$ and $x = \ln t$, $y = \cos t$.

Answers: 1) $w_x = 1/(x + 2y + 3z)$, $w_y = 2/(x + 2y + 3z)$, $w_z = 3/(x + 2y + 3z)$. 2) $z_x = (3yz - 2x)/(2z - 3yx)$, $z_y = (3xz - 2y)/(2z - 3yx)$. 3) $x + y + z = 0$. 5) $z = x + 1$. 6) $f(x, y, z) \approx 60x + \frac{24}{5}y + \frac{32}{5}z - 120$. Plugging in $(1.98, 3.01, 3.97)$ for (x, y, z) gives approximately 38.656. 7) $[(x/t) - y \sin t]/\sqrt{1 + x^2 + y^2}$.