Math 53 Discussion–vector functions, lines and planes
Answers on the back.

1) Find the tangent vector function $\overrightarrow{r}'(t)$ for the curve $\overrightarrow{r}(t) = \langle t^2, \cos 2t, -te^{-t} \rangle$. Find the equation of the tangent line to the curve at $t = 0$.

2) [13.1, #43] Find the vector function that represents the curve of intersection of the two surfaces given by the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

3) [13.4, #13] Find the velocity, acceleration, and speed of a particle with the position function $\overrightarrow{r}(t) = e^t(\cos t\ \hat{\mathbf{i}} + \sin t\ \hat{\mathbf{j}} + t\ \hat{\mathbf{k}})$.

4) [13.4, #45] The position function of a spaceship is

$$\overrightarrow{r}(t) = (3+t)\hat{\imath} + (2+\ln t)\hat{\jmath} + \left(7 - \frac{4}{t^2+1}\right)\hat{k}$$

and the coordinates of a space station are $(6, 4, 9)$. The captain wants the spaceship to coast into the space station. When should the engines be turned off?

Answers: 1) $\overrightarrow{r}'(t) = \langle 2t, -2\sin 2t, e^{-t}(-1+t)\rangle$, parametric equation for the tangent line at $t = 0$ is $\langle 0, 1, -s\rangle$, where s is the parameter. 2) Look at the cylinder in the x, y plane as a circle $(\cos t, \sin t)$. Then find z from the hyberboloid. $\overrightarrow{r}(t) = \cos t\ \hat{\mathbf{i}} + \sin t\ \hat{\mathbf{j}} + \cos(2t)\ \hat{\mathbf{k}}$, $0 \le t \le 2\pi$. 3) $\overrightarrow{v}(t) = e^t\langle \cos t - \sin t, \sin t + \cos t, t+1\rangle$, $\overrightarrow{a}(t) = e^t\langle -2\sin t, 2\cos t, t+2\rangle$, $|\overrightarrow{v}(t)| = e^t\sqrt{t^2+2t+3}$. 4) $t = 1$.