Math 53 Discussion

Practice Problems (from textbook and Math 53 worksheets):
Area and arc length of parametric curves

1) When a parametric curve comes from the graph of a function $y = f(x)$ for $a \leq x \leq b$, show that the formula for arc-length gives

$$\int_a^b \sqrt{1 + f'(x)^2}\, dx$$

2) [Worksheet 2, Problem 1] Let C be the curve $x = 2\cos t, y = \sin t$.
- What kind of curve is this?
- Find the slope of the tangent line to the curve when $t = 0, t = \pi/4$, and $t = \pi/2$.
- Find the area of the region enclosed by C. (Hint: $\sin^2 t = (1 - \cos 2t)/2$.)

3) Find the area of the region enclosed by the curve $x(t) = 1 - t, y(t) = e^t$ and the vertical lines $x = 0$, $x = 2$.

4) Find the arc length of the curve $x(t) = e^t + e^{-t}, y(t) = 2t - 5$ for $0 \leq t \leq 3$.

Answers: 1) Use the parametrization $(t, f(t))$. 2) Ellipse. Slopes are infinite, $-1/2$, 0. Area is 2π; it helps to compute a quarter of the area and then multiply by 4. Note that the bounds on x always go from smaller x to larger x, however when we change variables to t using the parametrization, it may happen that the lower bound on t is larger than the upper bound on t. That's okay - we get a positive answer which is what we expect for area. 3) $\int_1^{-1} e^t(-dt) = e - \frac{1}{e}$. 4) $\int_0^3 \sqrt{(e^t - e^{-t})^2 + 4}\, dt = e^3 - e^{-3}$.