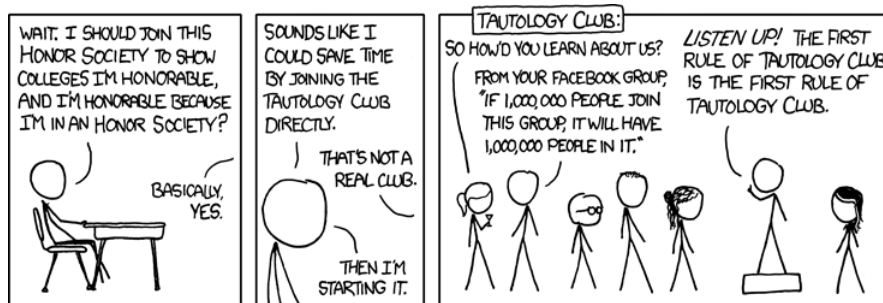


# Worksheet 9: Derivatives and Limits

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1. If  $g(x) = x^4 - 2$ , find  $g'(1)$  using the definition of the derivative and use it to find the equation of the tangent line of  $g(x)$  at  $(1, -1)$ .
2. List, with either an example graph or function and non-differentiable point, the ways in which a function can fail to be differentiable:
  - (a)
  - (b)
  - (c)
3. (True or False) and why:
  - (a) If a function is differentiable, then it is continuous.
  - (b) If a function is continuous, then it is differentiable.
4. Give the physics interpretation for each of the following:
  - (a) First Derivative
  - (b) Second Derivative
  - (c) Third Derivative
5. (True or False) and why.
  - (a) If  $\lim_{x \rightarrow 5} f(x) = 0$  and  $\lim_{x \rightarrow 5} g(x) = 0$ , then  $\lim_{x \rightarrow 5} \frac{f(x)}{g(x)}$  does not exist.

(b) If neither  $\lim_{x \rightarrow a} f(x)$  nor  $\lim_{x \rightarrow a} g(x)$  exists, then  $\lim_{x \rightarrow a} f(x) + g(x)$  does not exist.

(c) If the limit  $\lim_{x \rightarrow 6} f(x)g(x)$  exists, then the limit is  $f(6)g(6)$ .

(d) If  $p(x)$  is a polynomial, then the limit  $\lim_{x \rightarrow 6} p(x)$  is  $p(6)$ .

(e) If  $\lim_{x \rightarrow 0} f(x) = \infty$  and  $\lim_{x \rightarrow 0} g(x) = \infty$ , then  $\lim_{x \rightarrow 0} f(x) - g(x) = 0$ .

6. Solve:

(a)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$

(b)  $\lim_{x \rightarrow 1} e^{x^3 - x}$

(c)  $\lim_{x \rightarrow 3} \frac{\sqrt{x + 6} - x}{x^3 - 3x^2}$

(d)  $\lim_{x \rightarrow \pi^-} \ln(\sin(x))$

7. Sketch the graph of a function for which  $f(0) = 0$ ,  $f'(0) = -1$ ,  $f(1) = 0$ , and  $f'(1) = -1$ .

8. Write the general form for:

(a) The Power Rule

(b) The Constant Multiple Rule

(c) The Sum Rule

(d) The Difference Rule

9. Find the first and second derivative of:  $f(x) = 6x^{-\frac{8}{3}}$ . Express them in both major notations.

10. Find the first and second derivative of:  $f(x) = e^x - 5$ . Express them in both major notations.