1. Find the numbers at which \( f \) is discontinuous. At which of these numbers is \( f \) continuous from the right, from the left, or neither.

\[
f(x) = \begin{cases} 
  x + 1 & : x \leq 1 \\
  \frac{1}{x} & : 1 < x < 3 \\
  \sqrt{x - 3} & : x \geq 3
\end{cases}
\]

\( f \) is discontinuous at 1 and 3 since \( x + 1 \neq \frac{1}{x} \) at 1 and \( \frac{1}{x} \neq \sqrt{x - 3} \) at 3. By definition, \( f \) is continuous from the left at 1 since the limit exists and is equal to \( f(1) \) (\( \lim_{x \to 1^-} x + 1 = 2 = f(1) \)). Similarly for 3 from the right. \( f \) is not however continuous from the right at 1 or from the left at 3 since the one-sided limits are not equal to \( f(1) \) and \( f(3) \) respectively.

2. Use the intermediate value theorem to show that there is a root of \( f(x) = -e^x + 3 - 2x \) in the interval \((0, 1)\).

Note first that \( f(x) \) is continuous on \([0, 1]\) since it is an algebraic combination of functions which are continuous over this interval. Note further that \( f(0) \neq f(1) \) and that \( f(0) = 2 > 0 > f(1) = 1 - e \). It follows by the intermediate value theorem that \( f(x) = 0 \) somewhere on the interval \((0, 1)\).

3. Find a constant \( c \) such that

\[
g(x) = \begin{cases} 
  x^2 - c^2 & : x < 4 \\
  cx + 20 & : x \geq 4
\end{cases}
\]

is continuous.

Note that \( g(x) \) will be continuous if the two functions given are equal at 4. Thus,

\[4^2 - c^2 = 4c + 20\]
\[16 - c^2 = \]
\[c^2 = 4c + 4\]
\[0 = c^2 + 4c + 4\]
\[0 = (c + 2)(c + 2)\]

And thus, \( c = -2 \).

4. Sketch a function \( f(x) \) such that \( \lim_{x \to 3} f(x) = -\infty \), \( \lim_{x \to \infty} f(x) = 2 \), \( f(0) = 0 \), and \( f \) is even.
5. Evaluate \( \lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1} \)

\[
\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1} = \lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1} \left( \frac{\frac{1}{x^2}}{x^2} \right) \\
= \lim_{x \to \infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{\frac{5}{x^2} + \frac{4}{x} + \frac{1}{x^2}} \\
= \frac{3}{5}
\]

6. Evaluate \( \lim_{x \to \infty} \sqrt{x^2 - 3} - x \)

\[
\lim_{x \to \infty} \sqrt{x^2 - 3} - x = \lim_{x \to \infty} \sqrt{x^2 - 3} - x \left( \frac{\sqrt{x^2 - 3} + x}{\sqrt{x^2 - 3} + x} \right) \\
= \lim_{x \to \infty} \frac{x^2 - 3 - x^2}{\sqrt{x^2 - 3} + x} \\
= \lim_{x \to \infty} \frac{-3}{\sqrt{x^2 - 3} + x} \\
= 0
\]