Worksheet 6: $\epsilon - \delta$ Limits and More!

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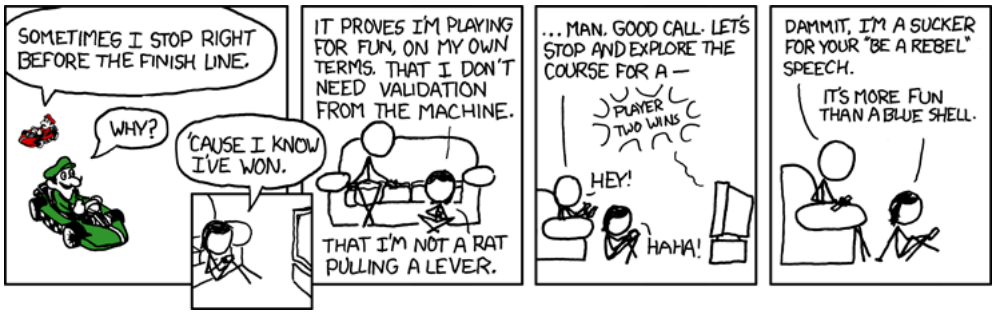

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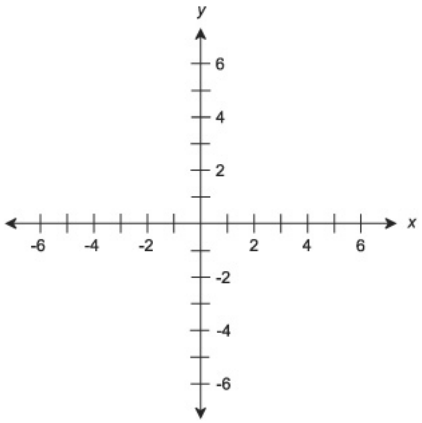
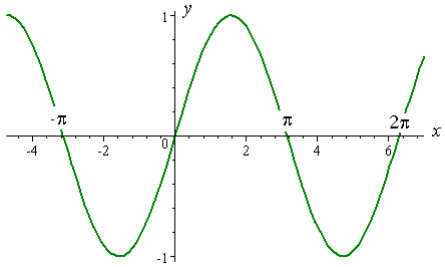
1. Let $\epsilon = \frac{1}{10}$. Use the graph below to find a δ that satisfies $|x - 3| < \delta \Rightarrow |f(x) - 3| < \epsilon$.



2. Let $\epsilon = \frac{7}{10}$. Find a δ that satisfies $|x - \frac{\pi}{2}| < \delta \Rightarrow |f(x) - 1| < \epsilon$ using $f(x) = \sin(x)$ and the graph below.



3. Let $f(x) = 12x$. Show that $\lim_{x \to 12} f(x) = 144$ using the $\epsilon - \delta$ definition of limit.

4. Let $f(x) = x^2$. Show that $\lim_{x \to 0} f(x) = 0$ using the $\epsilon - \delta$ definition of limit.

5. Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither.

$$f(x) = \begin{cases} x+1 & : x \leq 1 \\ \frac{1}{x} & : 1 < x < 3 \\ \sqrt{x-3} & : x \geq 3 \end{cases}$$

6. Use the intermediate value theorem to show that there is a root of $f(x) = -e^x + 3 - 2x$ in the interval $(0, 1)$.

7. Find a constant c such that

$$g(x) = \begin{cases} x^2 - c^2 & : x < 4 \\ cx + 20 & : x \geq 4 \end{cases}$$

is continuous.

8. Sketch a function $f(x)$ such that $\lim_{x \to 3} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = 2$, $f(0) = 0$, and f is even.