Worksheet 5: Not PreCalc!

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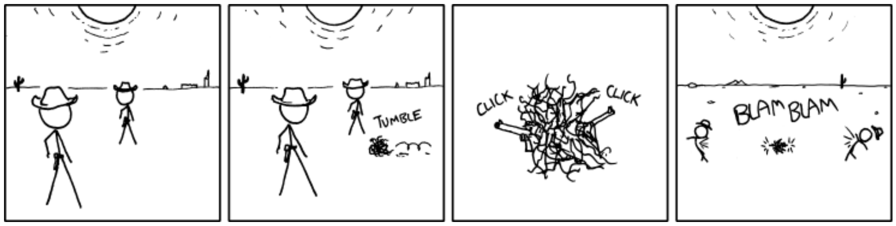

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1. Determine the infinite limit: $\lim_{x\to -3^-} \frac{x+2}{x+3}$.

   Note that, since we approach -3 from the left, we're dealing with values that are slightly more negative than -3. Because of this, the bottom of the fraction given $(x+3)$ is negative along with the top $(x+2)$. The overall fraction is therefore positive, and thus the limit approaches ∞.

2. Evaluate the limit: $\lim_{t\to 2} \frac{t^2-2}{t^3-3t+5}$.

   Because the function given is rational and isn't undetermined, we may simply plug in the value x is approaching: $\frac{2^2-2}{2^3-3(2)+5} = \frac{2}{8-6+5} = \frac{2}{7}$.

3. Evaluate the limit, if it exists:

   (a) $\lim_{t\to 1} \frac{t^4-1}{t^3-1}$.

$$\begin{aligned}
\lim_{t\to 1} \frac{t^4-1}{t^3-1} &= \lim_{t\to 1} \frac{(t^2-1)(t^2+1)}{(t-1)(t^2+t+1)} \\
&= \lim_{t\to 1} \frac{(t-1)(t+1)(t^2+1)}{(t-1)(t^2+t+1)} \\
&= \lim_{t\to 1} \frac{(t+1)(t^2+1)}{(t^2+t+1)} \\
&= \frac{(2)(2)}{1+1+1} \\
&= \frac{4}{3}
\end{aligned}$$

   (b) $\lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t}$.

$$\begin{aligned}
\lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t} &= \lim_{t\to 0} \frac{1}{t\sqrt{1+t}} - \frac{\sqrt{1+t}}{t\sqrt{1+t}} \\
&= \lim_{t\to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}} \\
&= \lim_{t\to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}} \left(\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}\right) \\
&= \lim_{t\to 0} \frac{1-(1+t)}{t\sqrt{1+t}(1+\sqrt{1+t})} \\
&= \lim_{t\to 0} \frac{-t}{t\sqrt{1+t}+t(1+t)} \\
&= \lim_{t\to 0} \frac{-1}{\sqrt{1+t}+1+t} \\
&= \frac{-1}{\sqrt{1}+1} \\
&= -1
\end{aligned}$$

(c) $\lim_{x \to -6} \dfrac{2x + 12}{|x + 6|}$.

Thinking about the function given in terms of the left and right limits or drawing a sketch of the function shows that the limit above is undefined.

4. The formal definition of the limit is:

$$\lim_{x \to a} f(x) = L \text{ if and only if}$$

$$\forall \epsilon > 0,\ \exists \delta \text{ such that if } 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \epsilon$$

Explain the second line of the definition in simple English.

For every ϵ greater than zero, there exists some δ such that whenever $0 < |x - a| < \delta$ holds, so does $|f(x) - L| < \epsilon$.