1. Determine the infinite limit: \( \lim_{x \to -3} \frac{x + 2}{x + 3} \)

Note that, since we approach \(-3\) from the left, we’re dealing with values that are slightly more negative than \(-3\). Because of this, the bottom of the fraction given \((x + 3)\) is negative along with the top \((x + 2)\). The overall fraction is therefore positive, and thus the limit approaches \(\infty\).

2. Evaluate the limit: \( \lim_{t \to 2} \frac{t^2 - 2}{t^3 - 3t + 5} \).

Because the function given is rational and isn’t undetermined, we may simply plug in the value \(x\) is approaching:

\[
\lim_{t \to 2} \frac{t^2 - 2}{t^3 - 3t + 5} = \frac{2^2 - 2}{2^3 - 3(2) + 5} = \frac{2}{8-6+5} = \frac{2}{7}.
\]

3. Evaluate the limit, if it exists:

(a) \( \lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1} \).

\[
\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1} = \lim_{t \to 1} \frac{(t^2 - 1)(t^2 + 1)}{(t - 1)(t^2 + t + 1)} = \lim_{t \to 1} \frac{(t - 1)(t + 1)(t^2 + 1)}{(t - 1)(t^2 + t + 1)} = \lim_{t \to 1} \frac{(t + 1)(t^2 + 1)}{(t^2 + t + 1)} = \frac{2(2)}{1+1+1} = \frac{4}{3}.
\]

(b) \( \lim_{t \to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \).

\[
\lim_{t \to 0} \frac{1}{t\sqrt{1+t}} - \frac{1}{t} = \lim_{t \to 0} \frac{1}{t\sqrt{1+t}} - \frac{\sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t \to 0} \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t \to 0} \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \left( \frac{1 + \sqrt{1+t}}{1 + \sqrt{1+t}} \right) = \lim_{t \to 0} \frac{1 - (1+t)}{t\sqrt{1+t}(1 + \sqrt{1+t})} = \lim_{t \to 0} \frac{1}{t\sqrt{1+t}(1 + \sqrt{1+t})} = \lim_{t \to 0} \frac{-1}{\sqrt{1+t}+1+t} = -1.
\]
(c) \[ \lim_{x \to -6} \frac{2x + 12}{|x + 6|}. \]

Thinking about the function given in terms of the left and right limits or drawing a sketch of the function shows that the limit above is undefined.

4. The formal definition of the limit is:

\[ \lim_{x \to a} f(x) = L \text{ if and only if } \forall \epsilon > 0, \exists \delta \text{ such that if } 0 < |x - a| < \delta \text{, then } |f(x) - L| < \epsilon \]

Explain the second line of the definition in simple English.

For every \( \epsilon \) greater than zero, there exists some \( \delta \) such that whenever \( 0 < |x - a| < \delta \) holds, so does \( |f(x) - L| < \epsilon \).