Worksheet 4: More PreCalc?!

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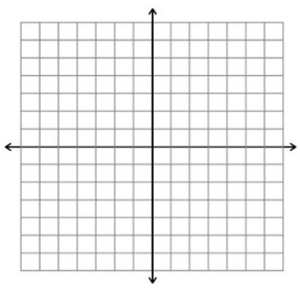
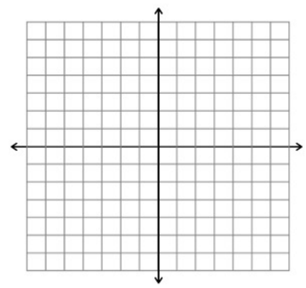
1. Are the following functions one-to-one? If so and represented with a graph, draw their inverse.

   (a) Yes

   (b) No

   (c) No

Sarah	Apple
Mike	Video Games
Angel	Guitar
Aun	Drums
Sophie	Guitar

2. What is the domain of the function $f(x) = ln(x^2 - 6x + 9)$?

   Recall that ln is only defined for values greater than 0. It follows, then, that we need to determine when $x^2 - 6x + 9$ is greater than 0. Graphing $x^2 - 6x + 9 = (x - 3)^2$ gives an upward parabola that touches the x-axis at $x = 3$, but never drops below it. The only point in the range of $x^2 - 6x + 9$ which is not greater than 0 is therefore $x = 3$, and so the domain for $f(x)$ is $\mathbb{R} - \{3\}$ or $(-\infty, 3) \cup (3, \infty)$.

3. Solve: $\frac{1}{x-5} < 7$.

   Although this problem looks simple, a correct solution requires considering two different cases since inequalities change with the multiplication of a negative number. In particular, we consider:

   Case 1: $x > 5$

   $$\begin{aligned} \frac{1}{x-5} &< 7 \\ 1 &< 7(x-5) \\ \frac{1}{7} &< x - 5 \\ x &> \frac{1}{7} + 5 \end{aligned}$$

   Case 2: $x < 5$

   $$\begin{aligned} \frac{1}{x-5} &< 7 \\ 1 &> 7(x-5) \\ \frac{1}{7} &> x - 5 \\ x &< \frac{1}{7} + 5 \end{aligned}$$

   But, by assumption, $x < 5$, so the overlap between the two inequalities or $x < 5$ is the solution.

   The final solution is therefore $x < 5$ or $x > \frac{1}{7} + 5$.

4. Find the domain: $f(x) = \sqrt{3 - \sqrt{x - 2}}$

   The square roots in this problem give rise to two different constraints:

   $$3 - \sqrt{x - 2} \geq 0$$

   $$x - 2 \geq 0$$

Solving the latter first, we obtain:

$$x \geq 2$$

Solving the former:

$$3 \geq \sqrt{x-2}$$
$$9 \geq x-2$$
$$11 \geq x$$

And so the final solution is $2 \leq x \leq 11$.

5. Sketch the graph of $f(x) = |x^2 - 2x|$.

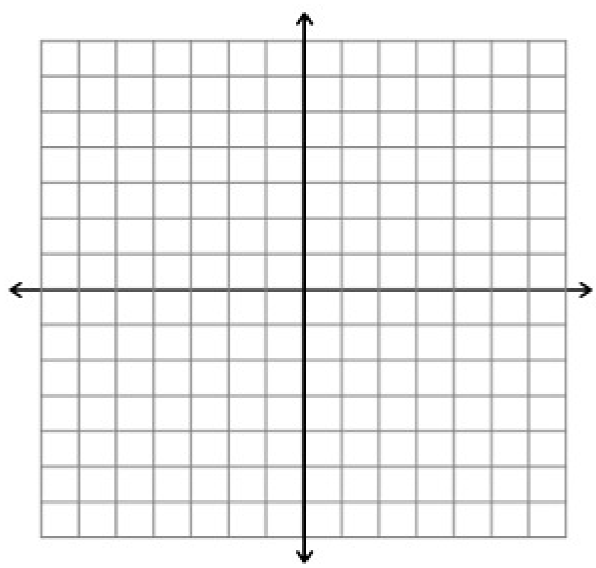
   We begin by noting that $x^2 - 2x$ is the equation for an upright parabola with zeroes at 0 and 2 ($x^2 - 2x = x(x-2)$). The sketch of the graph follows easily.

6. For the function $f(x)$ graphed below, give the following:

   (a) $\lim_{x \to 2} f(x) = -\infty$ (b) $\lim_{x \to 5} f(x) = \infty$

   (c) $\lim_{x \to -3^-} f(x) = -\infty$ (d) $\lim_{x \to -3^+} f(x) = \infty$

   (d) The vertical asymptotes: $x = -3, x = 2$, and $x = 5$.



7. Draw a function $f(x)$ such that: $\lim_{x \to 2^-} f(x) = -2$, $\lim_{x \to 2^+} f(x) = 0$, and $f(2) = 2$.