1. If I want to find the area over \([a, b]\) bounded above and below by \(f(x)\) and \(g(x)\) respectively where both are continuous over the interval and \(f(x) > g(x) > 0\) over the interval, what would I use for an integral? This is simply the area below \(f(x)\) minus the area below \(g(x)\); noting that integrals play nicely with subtraction,
\[
\int_{a}^{b} (f(x) - g(x))\,dx
\]

2. If I want to find the area over \([a, b]\) bounded above and below by \(f(x)\) and \(g(x)\) respectively where both are continuous over the interval, what would I use for an integral? How would I solve for the integral? Since we don’t have any idea how \(f(x)\) relates to \(g(x)\),
\[
\int_{a}^{b} |f(x) - g(x)|\,dx
\]

3. Sketch the region enclosed by the given curves and find its area:
   (a) \(y = |x|, y = x^2 - 2\)

Looking at the graph, it’s natural to take the integral with respect to the \(x\)-axis and break it at \(x = 0\) to avoid dealing with the absolute value. Since \(y = |x|\) is always above \(y = x^2 - 2\), we need only find the intersections shown.

Breaking the absolute value,
\[
-x = x^2 - 2
\]
\[
x^2 + x - 2 = 0
\]
\[
(x - 1)(x + 2) = 0
\]
\[
x = 1, -2
\]

Noting this is the \(-x\) side, \(x = 1\) isn’t a viable option, so \(x = -2\) is the intersection. Similarly,
\[
x = x^2 - 2
\]
\[
x^2 - x - 2 = 0
\]
\[
(x + 1)(x - 2) = 0
\]
\[
x = -1, 2
\]

Noting this is the \(x\) side, \(x = -1\) isn’t a viable option, so \(x = 2\) is the other intersection. Thus, the relevant integral is
\[ \int_{-2}^{0} [-x - (x^2 - 2)] \, dx + \int_{0}^{2} [x - (x^2 - 2)] \, dx \]

Noting that the areas on each side of 0 are the same, we have

\[ 2 \int_{0}^{2} [x - (x^2 - 2)] \, dx \]
\[ 2 \int_{0}^{2} [-x^2 + x + 2] \, dx \]
\[ 2 \left( -\frac{1}{3} x^3 + \frac{1}{2} x^2 + 2x \right)_{0} \]
\[ 2 \left( -\frac{1}{3} x^3 + \frac{1}{2} x^2 + 4 \right) \]
\[ -\frac{2^4}{3} + 2^2 + 2^3 \]
\[ \frac{20}{3} \]

(b) \( x = y^4, y = \sqrt{2-x}, y = 0 \)

Looking at the graph, it's natural to take the integral with respect to the \( y \)-axis. Since \( y = \sqrt{2-x} \) is always 'above' \( x = y^4 \), we need only find the intersection shown (the other bound being 0).

Setting the two equal,

\[ y = \sqrt{2-x} \]
\[ y^2 = 2 - x \]
\[ y^4 + y^2 - 2 = 0 \]
\[ (y^2 + 2)(y^2 - 1) = 0 \]
\[ y = \pm 1, \pm \sqrt{2} \]

Noting that the intersection is a positive \( y \) value in the real plane, \( y = 1 \) is the desired intersection. Solving for \( y \),

\[ y = \sqrt{2-x} \]
\[ y^2 = 2 - x \]
\[ x = 2 - y^2 \]

Thus, the relevant integral is

\[ \int_{0}^{1} [2 - y^2 - y^4] \, dy \]
4. What formulas represent the volume of a solid of revolution?

\[ \int_{a}^{b} A(x) \, dx \]

\[ \int_{a}^{b} A(y) \, dy \]

where \([a, b]\) is the interval on the x/y axis enclosing the solid and \(A(x)/A(y)\) gives the area of a vertical/horizontal slice at \(x/y\) in the interval \([a, b]\).

5. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

(a) \(y = \frac{1}{4}x^2, y = 5 - x^2\); about the x-axis

Looking at the graph, it’s natural to take the integral with respect to the x-axis. We start by finding the interval over which we will integrate:

Setting the two equations equal,

\[ \frac{1}{4}x^2 = 5 - x^2 \]
\[ x^2 = 20 - 4x^2 \]
\[ 5x^2 = 20 \]
\[ x^2 = 4 \]
\[ x = \pm 2 \]

Next, we need an expression for \(A(x)\), the area at any \(x\)-slice of the solid. Drawing an arbitrary slice gives,

\[ A(x) \] is thus the area of the larger circle minus the area of the smaller, \(A(x) = \pi(5 - x^2)^2 - \pi\left(\frac{1}{4}x^2\right)^2\). The desired integral is thus,

\[ \int_{-2}^{2} [\pi(5 - x^2)^2 - \pi\left(\frac{1}{4}x^2\right)^2] \, dx \]
\[ \int_{-2}^{2} \pi[25 - 10x^2 + x^4 - \frac{1}{16}x^4] \, dx \]
\[
\begin{align*}
\pi \int_{-2}^{2} \left[ 25 - 10x^2 + \frac{15}{16} x^4 \right] dx \\
\pi \left( 25x - \frac{10}{3} x^3 + \frac{15}{80} x^5 \right)_{-2} \\
2\pi \left( 25(2) - \frac{10}{3} 2^3 + \frac{15}{80} 2^5 \right) \\
2\pi \left( 50 - \frac{10}{3} 2^3 + 6 \right) \\
\frac{176\pi}{3}
\end{align*}
\]

(b) \( y = \frac{1}{4} x^2, x = 2, y = 0; \) about the \( y \)-axis

\[
\int_{-2}^{2} \pi (4 - 4y) \, dy
\]

Looking at the graph, it's natural to take the integral with respect to the \( y \)-axis (an \( x \)-slice is not a rectangle...). We start by noting that the interval over which we will integrate is \([-2,2]\). Next, we need an expression for \( A(x) \), the area at any \( y \)-slice of the solid. Drawing an arbitrary slice gives,

\[
A(y) \text{ is thus the area of the larger circle minus the smaller, } A(y) = \pi (2)^2 - \pi (2\sqrt{y})^2 = \pi (4 - 4y). \text{ The desired integral is thus,}
\]

\[
\int_{-2}^{2} \pi (4 - 4y) \, dy
\]

(c) \( y = e^{-x}, y = 1, x = 2; \) about \( y = 2 \)
Looking at the graph, it’s natural to take the integral with respect to the $x$-axis. We start by noting that the interval over which we will integrate is $[0,2]$. Next, we need an expression for $A(x)$, the area at any $x$-slice of the solid. Drawing an arbitrary slice gives,

$$A(x) = \pi (2 - e^{-x})^2 - \pi (1)^2 = \pi (2 - e^{-x})^2 - \pi.$$ The desired integral is thus,

$$\int_{0}^{2} \left[ \pi (2 - e^{-x})^2 - \pi \right] dx$$

$$\pi \int_{0}^{2} \left[ 4 - 4e^{-x} + e^{-2x} - 1 \right] dx$$

$$\pi \int_{0}^{2} \left[ 3 - 4e^{-x} + e^{-2x} \right] dx$$

$$\pi \left[ 3x + 4e^{-x} - \frac{1}{2} e^{-2x} \right]_0^2$$

$$\pi \left[ 6 + 4e^{-2} - \frac{1}{2} e^{-4} - (4 - \frac{1}{2}) \right]$$

$$\pi \left[ \frac{5}{2} + 4e^{-2} - \frac{1}{2} e^{-4} \right]$$

$$\pi (5e^4 + 8e^2 - 1)$$

6. Use the shell method to find the volume of the solid obtained by rotating the region bounded by the given curves about the $y$-axis.

(a) $y = x^3$, $y = 0$, $x = 1$, $x = 2$

Looking at the graph, notice that the interval over which we will integrate is $[1,2]$. Since $f(x) = x^3$ is already given, and we wish to find the volume of a solid rotated around the $y$-axis, we meet all the conditions for the shell method and can simply plug in our values to get the necessary integral:

$$\int_{1}^{2} (2\pi x)(x^3) \, dx$$
\[
\int_{1}^{2} 2\pi x^4 \, dx
\]
\[
= \left. \frac{2\pi}{5} x^5 \right|_{1}^{2}
\]
\[
= \frac{2\pi}{5} 2^5 \quad \frac{2\pi}{5} 1^5
\]
\[
= \frac{2\pi}{5} (2^5 - 1)
\]