Worksheet 3: Even More PreCalc!

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1. What is the domain of $ln(x)$? What is $ln(1)$?

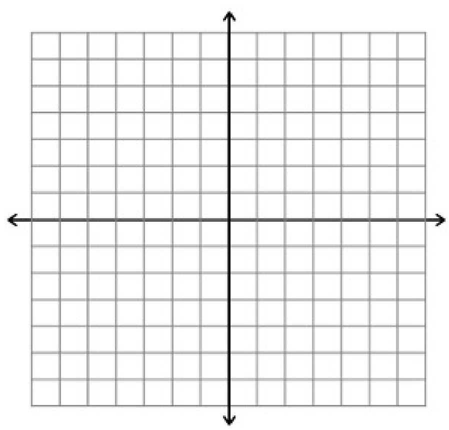
   The domain of $ln(x)$ is $(0, \infty)$. $ln(1) = 0$ as it does for every logarithm.

2. Is $f(x) = x^2 + 4x + 4$ injective? Is it one-to-one?

   $f(x)$ is not injective (it's a parabola and therefore fails the horizontal line test). It is also not one-to-one since one-to-one is the same thing as injective.

3. Find the exponential function $f(x) = Ca^x$ matching the graph below.

   By plugging in the given points, we obtain: $f(-1) = Ca^{-1} = \frac{C}{a} = 3$ and $f(1) = Ca = \frac{4}{3}$.



Solving for C in the former,

$$C = 3a$$

Substituting into the latter,

$$3a(a) = \frac{4}{3}$$
$$3a^2 =$$
$$a^2 = \frac{4}{9}$$

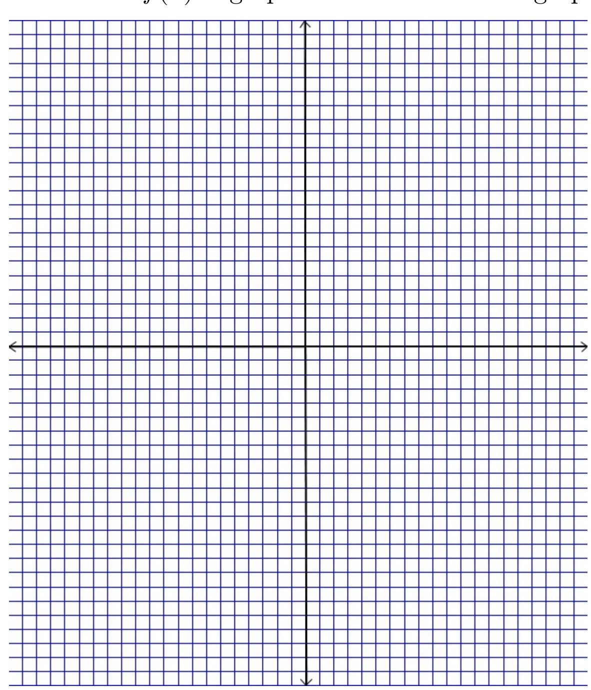
Recalling that $a > 0$ by definition,

$$a = \sqrt{\frac{4}{9}}$$
$$= \frac{2}{3}$$

And thus,

$$C = 2$$

4. A function $f(x)$ is graphed below. Draw the graphs of $f(\frac{1}{2}x - 4)$ and $2(f(x) + 1)$.



Consider first $f(\frac{1}{2}x - 4)$. It's always helpful to separate the transformations, so $f(\frac{1}{2}x - 4) = f(\frac{1}{2}(x - 8))$. The graph therefore undergoes a rightward shift of 8 units $(x - 8)$ and a horizontal stretch by 2 (from the $\frac{1}{2}$ in front; the graph now moves half as fast along the x-axis).

Consider now $2(f(x) + 1)$. Again, it's helpful to separate the transformations: $2(f(x) + 1) = 2f(x) + 2$. This means that the graph is shifted upward by 2 units (+2) while also being stretched vertically by a factor of 2. As a bit of advice, do the stretch first so that you know which points should and shouldn't be impacted (points on the x-axis aren't affected; all others are).

5. Moved to next worksheet.

6. Let $f(x) = ln(x+3)$. Find $f^{-1}(x)$.

$$\begin{aligned} f(x) = \ln(x+3) \rightarrow x &= \ln(f^{-1}(x)+3) \\ e^x &= e^{\ln(f^{-1}(x)+3)} \\ e^x &= f^{-1}(x)+3 \\ e^x - 3 &= f^{-1}(x) \\ f^{-1}(x) &= e^x - 3 \end{aligned}$$

7. Simplify: $ln(e^{2sin(x)}e^{2cos(x)}) + \frac{9^x}{3^{2x}} - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2$

$$\begin{aligned} ln(e^{2sin(x)}e^{2cos(x)}) + \frac{9^x}{3^{2x}} - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2 &= ln(e^{2sin(x)+2cos(x)}) + \frac{9^x}{3^{2x}} - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2 \\ &= 2sin(x) + 2cos(x) + \frac{9^x}{3^{2x}} - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2 \\ &= 2sin(x) + 2cos(x) + \frac{9^x}{9^x} - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2 \\ &= 2sin(x) + 2cos(x) + 1 - \left(\frac{cot(x)sin(x)}{cos(x)}\right)^2 \\ &= 2sin(x) + 2cos(x) + 1 - \left[\left(\frac{cos(x)}{sin(x)}\right)\left(\frac{sin(x)}{cos(x)}\right)\right]^2 \\ &= 2sin(x) + 2cos(x) + 1 - 1 \\ &= 2(sin(x) + cos(x)) \end{aligned}$$

8. Moved to next worksheet.

9. Find the exact value of $\log_5(25) + \log_{10}(1000) + \ln(e^7)$.
   $\log_5(5^2) + \log_{10}(10^3) + \ln(e^7) = 2 + 3 + 7 = 12$

10. Find the exact value of $\log_3(15) + \log_3(12) - \log_3(20)$.

$$\begin{aligned} \log_3(15) + \log_3(12) - \log_3(20) &= \log_3\left(\frac{(15)(12)}{20}\right) \\ &= \log_3\left(\frac{3^2 \cdot 5 \cdot 4}{5 \cdot 4}\right) \\ &= \log_3(3^2) \\ &= 2 \end{aligned}$$

11. Find the exact value of $\log_{42}(-42)$.
    Does not exist.