1. What is the domain of $\ln(x)$? What is $\ln(1)$?
   The domain of $\ln(x)$ is $(0, \infty)$. $\ln(1) = 0$ as it does for every logarithm.

2. Is $f(x) = x^2 + 4x + 4$ injective? Is it one-to-one?
   $f(x)$ is not injective (it’s a parabola and therefore fails the horizontal line test). It is also not one-to-one since one-to-one is the same thing as injective.

3. Find the exponential function $f(x) = Ca^x$ matching the graph below.
   By plugging in the given points, we obtain: $f(-1) = Ca^{-1} = \frac{C}{a} = 3$ and $f(1) = Ca = \frac{4}{3}$.
   Solving for $C$ in the former,
   $$C = 3a$$
   Substituting into the latter,
   $$3a(a) = \frac{4}{3}$$
   $$3a^2 = \frac{4}{a}$$
   $$a^2 = \frac{4}{9}$$
   Recalling that $a > 0$ by definition,
   $$a = \sqrt{\frac{4}{9}}$$
   $$= \frac{2}{3}$$
   And thus,
   $$C = 2$$

4. A function $f(x)$ is graphed below. Draw the graphs of $f(\frac{1}{2}x - 4)$ and $2(f(x) + 1)$.
   Consider first $f(\frac{1}{2}x - 4)$. It’s always helpful to separate the transformations, so $f(\frac{1}{2}x - 4) = f(\frac{1}{2}(x - 8))$. The graph therefore undergoes a rightward shift of 8 units $(x - 8)$ and a horizontal stretch by 2 (from the $\frac{1}{2}$ in front; the graph now moves half as fast along the x-axis).
   Consider now $2(f(x) + 1)$. Again, it’s helpful to separate the transformations: $2(f(x) + 1) = 2f(x) + 2$. This means that the graph is shifted upward by 2 units $(+2)$ while also being stretched vertically by a factor of 2. As a bit of advice, do the stretch first so that you know which points should and shouldn’t be impacted (points on the x-axis aren’t affected; all others are).

5. Moved to next worksheet.
6. Let \( f(x) = \ln(x + 3) \). Find \( f^{-1}(x) \).

\[
f(x) = \ln(x + 3) \rightarrow x = \ln(f^{-1}(x) + 3)
\]

\[
e^x = e^{\ln(f^{-1}(x) + 3)}
\]

\[
e^x = f^{-1}(x) + 3
\]

\[
e^x - 3 = f^{-1}(x)
\]

\[
f^{-1}(x) = e^x - 3
\]

7. Simplify: \( \ln(e^{2\sin(x)}e^{2\cos(x)}) + \frac{9x}{32x} - \left( \frac{\cot(x)\sin(x)}{\cos(x)} \right)^2 \)

\[
\ln(e^{2\sin(x)}e^{2\cos(x)}) + \frac{9x}{32x} - \left( \frac{\cot(x)\sin(x)}{\cos(x)} \right)^2 = \ln(e^{2\sin(x)+2\cos(x)}) + \frac{9x}{32x} - \left( \frac{\cot(x)\sin(x)}{\cos(x)} \right)^2
\]

\[
= 2\sin(x) + 2\cos(x) + \frac{9x}{32x} - \left( \frac{\cot(x)\sin(x)}{\cos(x)} \right)^2
\]

\[
= 2\sin(x) + 2\cos(x) + 1 - \left( \frac{\cot(x)\sin(x)}{\cos(x)} \right)^2
\]

\[
= 2\sin(x) + 2\cos(x) + 1 - \left[ \left( \frac{\cos(x)}{\sin(x)} \right) \left( \frac{\sin(x)}{\cos(x)} \right) \right]^2
\]

\[
= 2\sin(x) + 2\cos(x) + 1 - 1
\]

\[
= 2(\sin(x) + \cos(x))
\]

8. Moved to next worksheet.

9. Find the exact value of \( \log_5(25) + \log_{10}(1000) + \ln(e^7) \).

\[
\log_5(5^2) + \log_{10}(10^3) + \ln(e^7) = 2 + 3 + 7 = 12
\]

10. Find the exact value of \( \log_3(15) + \log_3(12) - \log_3(20) \).

\[
\log_3(15) + \log_3(12) - \log_3(20) = \log_3 \left( \frac{15(12)}{20} \right)
\]

\[
= \log_3 \left( \frac{3^2 \cdot 5 \cdot 4}{5 \cdot 4} \right)
\]

\[
= \log_3(3^2)
\]

\[
= 2
\]

11. Find the exact value of \( \log_{42}(-42) \).

Does not exist.