Worksheet 29: The Fundamental Thm. of Calculus

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1. The fundamental theorem of calculus has one assumption and two parts (see page. 393 if you don't remember).

   (a) What is the assumption?

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   (b) What are the two conclusions?

2. What, conceptually, is a function of the form $g(x) = \int_a^x f(t)\, dt$? How is x constrained?

3. Find the derivative of the following:

   (a) $g(x) = \int_3^x e^{t^2 - t}\, dt$

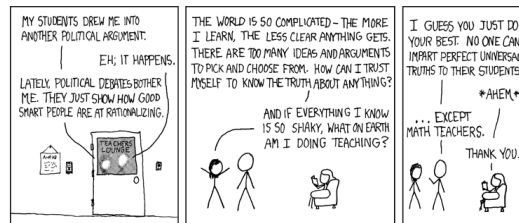   (b) $g(r) = \int_3^r \sqrt{x^2 + 4}\, dx$

   (c) $G(x) = \int_x^1 \cos(\sqrt{t})\, dt$

   (d) $y = \int_0^{x^4} \cos^2(\theta)\, d\theta$

4. Evaluate the integral

   (a) $\int_{-1}^1 x^{100}\, dx$

   (b) $\int_{-5}^5 e\, dx$

(c) $\int_0^{\frac{\pi}{4}} \sec(\theta) \tan(\theta) \, d\theta$

5. What's wrong?

$$\int_{-1}^{2} \frac{4}{x^3} \, dx = \left. \frac{-2}{x^2} \right]_{-1}^{2} = \frac{3}{2}$$

6. What's the difference between $\int_0^1 (x^3 + x + 1) \, dx$ and $\int x^3 + x + 1$? Find each.

7. Verify the following:

$$\int \cos^3(x) \, dx = \sin(x) - \frac{1}{3} \sin^3(x) + C$$

8. Find the general indefinite integral:

(a) $\int 2^x \, dx$

(b) $\int \sec(t)(\sec(t) + \tan(t)) \, dt$

9. Evaluate the integral:

(a) $\int_0^3 (1 + 6w^2 - 10w^4) dw$

(b) $\int_0^1 (5x - 5^x) dx$