Worksheet 28: Definite Integrals

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1. Determine two different regions whose area is equal to the limit $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\frac{2}{n}(5+\frac{2i}{n})^{10}$.

2. Give a summation for the area of the function $f(x)$ over the interval $[a, b]$.

3. Assuming that $\int_a^b f(x)dx$ exists, how does it relate to the solution in (2)?

4. Express the following as an integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}[(a+\frac{(b-a)i}{n})^3 + (a+\frac{(b-a)i}{n})\sin(a+\frac{(b-a)i}{n})](\frac{(b-a)}{n})$$

5. What is the value of $\int_b^a f(x)dx$ (in terms of $\int_a^b f(x)dx$)? Why?

6. What is the value of $\int_a^a f(x)dx$? Why?

7. Find an expression for the value of $\int_a^b c\,dx$ where c is any constant.

8. ($\star$) Use a Riemann sum representation with right endpoints to find a value for $\int_2^5 (4 - 2x)dx$

9. Evaluate the integral by interpreting it in terms of areas: $\int_{-1}^2 (1 - x)dx$

10. Show that $\int_a^b x\,dx = \frac{b^2 - a^2}{2}$