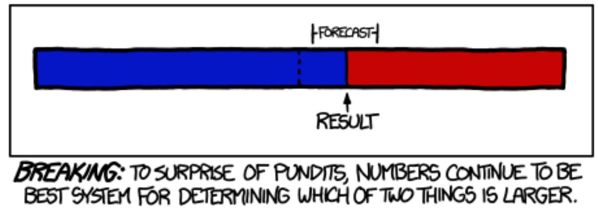
# Worksheet 25: Newton's Method


Russell Buehler

b.r@berkeley.edu


www.xkcd.com

1. Use Newton's method starting with $x_1 = -1$ to find $x_3$ the third approximation of the root of $x^7 + 4 = 0$.

   Recall that the formula for Newton's method is:

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

   Solving for $f'(x)$ generally,

$$f'(x) = 7x^6$$

   Thus,

$$\begin{aligned} x_2 &= -1 + \frac{f(-1)}{f'(-1)} \\ &= -1 + \frac{3}{7} \\ &= -\frac{4}{7} \end{aligned}$$

   Solving for $x_3$,

$$\begin{aligned} x_3 &= -\frac{4}{7} - \frac{f(-\frac{4}{7})}{f'(-\frac{4}{7})} \\ &= -\frac{4}{7} - \frac{-\frac{4^7}{7^7} + 4}{\frac{4^6}{7^5}} \\ &= -\frac{4}{7} + \frac{4}{7^2} - \frac{7^5}{4^5} \\ &= -\frac{24}{7^2} - \frac{7^5}{4^5} \end{aligned}$$

2. Use Newton's method to approximate $\sqrt[100]{100}$ to 4 decimal places.

$$\begin{aligned} \sqrt[100]{100} &= x \\ 100 &= x^{100} \\ x^{100} - 100 &= 0 \end{aligned}$$

   Let $f(x) = x^{1}00 - 100$. It follows immediately that $f'(x) = 100x^{99}$. We now employ Newton's method, starting with $x_1 = 1$.

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 1.99 \\ x_3 &= 1.9701 \\ x_4 &= 1.950399 \\ &\vdots \end{aligned}$$

3. Use Newton's method to find the roots of $\frac{1}{x} = 1 + x^3$ to 3 decimal places.

Let $f(x) = 1 + x^3 - \frac{1}{x}$. It follows immediately that $f'(x) = 3x^2 + x^{-2}$. We now employ Newton's method, starting with $x_1 = 1$.

$$x_1 = 1$$
$$x_2 = .75$$
$$x_3 = .72444$$
$$x_4 = .72449$$

4. Find the most general anti-derivative:

   (a) $f(x) = \frac{1}{2}x^2 - 2x + 6$

   $$F(x) = \frac{1}{6}x^3 - x^2 + 6x + C$$

   (b) $f(x) = x(2-x)^2 = x(4 - 4x + x^2) = 4x - 4x^2 + x^3$

   $$F(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + C$$

   (c) $y = e^2$

   $$F(x) = e^2 x + C$$

   (d) $f(x) = \sqrt[3]{x^2} + x\sqrt{x} = x^{\frac{2}{3}} + x^{\frac{3}{2}}$

   $$F(x) = \frac{3}{5}x^{\frac{5}{3}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

   (e) $r(\theta) = \sec(\theta)\tan(\theta) - 2e^\theta$

   $$R(\theta) = \tan(\theta) - 2e^\theta + C$$

   (f) $f(x) = \frac{2x}{1+x^2}$

   $$F(x) = \ln(1 + x^2) + C$$

5. Find $f$

   (a) $f''(x) = 8x^3 + 5, f(1) = 0, f'(1) = 8$
   See next worksheet

   (b) $f''(t) = 2e^t + 3\sin(t), f(0) = 0, f'(\pi) = 0$
   See next worksheet

   (c) $f'''(x) = \cos(x), f(0) = 1, f'(0) = 2, f''(0) = 3$
   See next worksheet