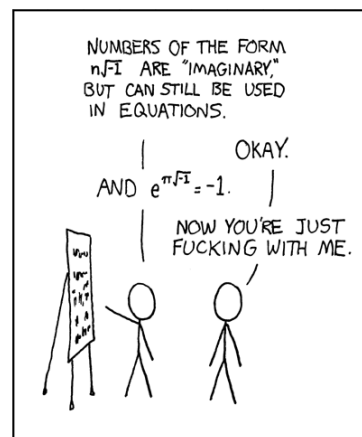# Worksheet 24: Optimization

Russell Buehler

b.r@berkeley.edu

www.xkcd.com

1. Let $P = \frac{100I}{I^2 + I + 4}$. For what values of I is P maximum?

2. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

3. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.

4. ($\star$) Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5. A cone-shaped drinking cup is to hold $27cm^3$ of water. Find the height and radius of the cup that will use that smallest amount of paper.

6. At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

7. (a) Show that if the profit $P(x)$ is a maximum, then the marginal revenue equals the marginal cost.

   (b) If $c(x) = 16,000 + 500x - 1.6x^2 + .004x^3$ is the cost function and $p(x) = 1700 - 7x$ is the demand function, find the production level that will maximize profit.