Worksheet 23: Bernoulli's Rule

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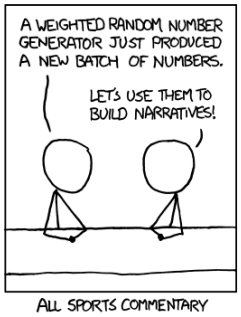

www.xkcd.com

1. State the assumptions of Bernoulli's (l'Hospital's) Rule.

2. Find the value of the following limits:

   (a) $\lim_{x\to\infty} \frac{e^x}{x^2}$

   (b) $\lim_{x\to 0^+} x\ln(x)$

   (c) $\lim_{x\to \frac{\pi}{2}^-} \sec(x) - \tan(x)$

   (d) $\lim_{x\to 0^+} x^x$

   (e) $\lim_{x\to\infty} \sqrt{x}e^{-\frac{x}{2}}$

3. Show that

$$\lim_{x\to\infty} \frac{\ln(x)}{x^p} = 0$$

for any number $p > 0$. This proves that the logarithmic function approaches ∞ more slowly than any positive power of x.

4. ($\star$) True or False; justify your answer

   (a) If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

   (b) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of $f(x)$.

   (c) There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.

   (d) There exists a function f such that $f(1) = -2$, $f(3) = 0$, and $f'(x) > 1$ for all x.

   (e) If f,g are increasing on an interval I, $f + g$ is increasing on I.

5. Sketch $f(x) = \sqrt[3]{x^3 - x}$ showing: increasing, decreasing, zeroes, behavior for $|x|$ large, behavior for $|x|$ small, and points where the function is not differentiable. You need not show convexity or points of inflection.

6. In section 4.5, Stewart gives a list of seven main attributes of functions which should be taken into account when sketching a curve; list them.