Worksheet 22: Concavity

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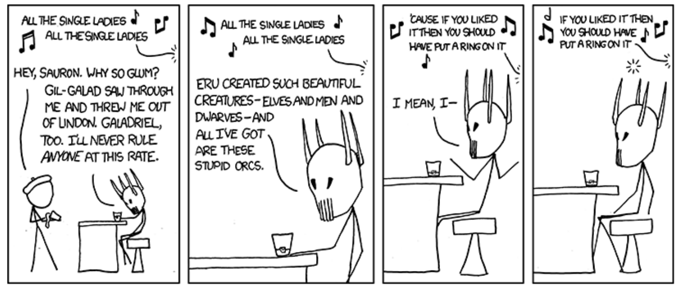

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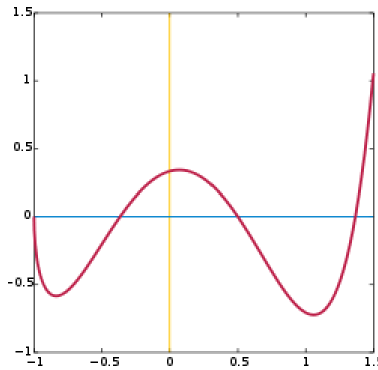
1. If you haven't already, label the local maxima/minima, absolute maximum/minimum, inflection points, and where the graph is concave up or concave down.



2. What is the first derivative test? What is the second derivative test?

3. If you haven't already, for $f(x) = 4x^3 + 3x^2 - 6x + 1$, find the intervals on which f is increasing or decreasing, the local maxima and minima, and the inflection points.

4. Show that the equation $x^3 - 15x + c$ has at most one root in the interval $[-2, 2]$.

5. Find the intervals on which f is increasing/decreasing and the local maxima and minima values for $f(x) = x - 2\sin(x)$, $0 < x < 3\pi$.

6. Sketch the graph of a function that satisfies all of the following: $f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$, $f'(x) > 0$ if $1 < |x| < 2$, $f'(x) = -1$ if $|x| > 2$, $f''(x) < 0$ if $-2 < x < 0$, inflection point at $(0, 1)$.

7. Find all critical numbers of $5x^{\frac{2}{3}} + x^{\frac{5}{3}}$.

8. Find the intervals on which f is increasing/decreasing and the local maxima and minima values for $f(x) = xe^x$

9. Sketch $f(x) = \sqrt[3]{x^3 - x}$ showing: increasing, decreasing, zeroes, behavior for $|x|$ large, behavior for $|x|$ small, and points where the function is not differentiable. You need not show convexity or points of inflection.