1. Verify that \( f(x) = x^3 - x^2 - 6x + 2 \) satisfies the hypotheses of Rolle’s theorem for the interval \([0, 3]\), then find all \( c \) that satisfy the conclusion.

2. Let \( f(x) = \tan(x) \). Show that \( f(0) = f(\pi) \), but there is no number \( c \) in \((0, \pi)\) such that \( f'(c) = 0 \). Is this a counterexample to Rolle’s theorem? Why or why not?

3. Verify that \( f(x) = x^3 - 3x + 2 \) satisfies the hypotheses of the mean value theorem on \([-2, 2]\), then find all \( c \) that satisfy the conclusion.

4. Let \( f(x) = \frac{x^3-x^2}{x-1} \) on \([0, 2]\). Show that there is no value of \( c \) such that \( f'(c) = \frac{f(2)-f(0)}{2-0} \). Is this a counterexample to the mean value theorem? Why or why not?

5. (⋆) If for two functions \( f(x) \) and \( g(x) \), we know that \( f'(x) = g'(x) \) for every \( x \) in an interval \((a, b)\), it must be the case that \( f - g \) is constant on \((a, b)\). Why? What can we say about \( f(x) \) in terms of \( g(x) \)?
6. (*) Using the mean value theorem and Rolle’s theorem, show that $x^3 + x - 1 = 0$ has exactly one real root.

7. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.

8. (a) Suppose that $f$ is differentiable on $\mathbb{R}$ and has two roots. Show that $f'$ has at least one root.

(b) Suppose $f$ is twice differentiable on $\mathbb{R}$ and has three roots. Show that $f''$ has at least one real root.

(c) Can you generalize parts (a) and (b)?

9. Label the local maxima/minima, absolute maximum/minimum, inflection points, and where the graph is concave up or concave down.
10. What is the first derivative test? What is the second derivative test?

11. For \( f(x) = 4x^3 + 3x^2 - 6x + 1 \), find the intervals on which \( f \) is increasing or decreasing, the local maxima and minima, and the inflection points.