Worksheet 2: More PreCalc!

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1. Let $f(x) = x^4$. Find $f(2)$, $f(4a)$, and $f(a-5)$.

   $f(2) = 2^4 = 16$

   $f(4a) = (4a)^4 = 256a^4$

   $f(a-5) = (a-5)^4$

2. Let $f(x) = -x^2 + 5x + 11$. Find $2f(a), f(2a), f(a^2), f(a)^2$, and $f(a+h)$.

$$\begin{aligned}
2f(a) &= 2(-a^2 + 5a + 11) = -2a^2 + 10a + 22 \\
f(2a) &= -(2a)^2 + 5(2a) + 11 = -4a^2 + 10a + 11 \\
f(a^2) &= -(a^2)^2 + 5(a^2) + 11 = -a^4 + 5a^2 + 11 \\
f(a)^2 &= (-a^2 + 5a + 11)^2 \\
f(a+h) &= -(a+h)^2 + 5(a+h) + 11
\end{aligned}$$

3. Let $f(x) = \frac{x+3}{x+1}$. Find $\frac{f(x)-f(1)}{x-1}$.

$$\begin{aligned}
\frac{f(x)-f(1)}{x-1} &= \frac{\frac{x+3}{x+1} - \frac{1+3}{1+1}}{x-1} \\
&= \frac{\frac{x+3}{x+1} - 2}{x-1} \\
&= \frac{\frac{x+3}{x+1} - \frac{2(x+1)}{x+1}}{x-1} \\
&= \frac{\frac{x+3-2x-2}{x+1}}{x-1} \\
&= \frac{\frac{-x+1}{x+1}}{x-1} \\
&= \frac{-(x-1)}{x+1}\left(\frac{1}{x-1}\right) \\
&= \frac{1}{x+1}
\end{aligned}$$

4. Explain the difference between something failing to be a function because of the 'Vertical Line Test' and failing because a single x-value was mapped to multiple y-values.

   There is no difference.

5. Classify, with justification, whether the following functions are even or odd.

(a) $f(x) = x^2$
Even;

$$\begin{aligned} f(x) &= x^2 \\ &= (-x)^2 \\ &= f(-x) \end{aligned}$$

(b) $f(x) = x^3 + x$
Odd;

$$\begin{aligned} -f(x) &= -(x^3 + x) \\ &= -x^3 - x \\ &= (-x)^3 - x \\ &= f(-x) \end{aligned}$$

(c) $f(x) = x^3 + 1$
Neither;

$$\begin{aligned} f(x) &= x^3 + 1 \\ f(-x) &= (-x)^3 + 1 \\ &= -x^3 + 1 \end{aligned}$$

$$f(x) \neq f(-x)$$

$$\begin{aligned} -f(x) &= -(x^3 + 1) \\ &= -x^3 - 1 \\ f(-x) &= (-x)^3 + 1 \\ &= -x^3 + 1 \end{aligned}$$

$$-f(x) \neq f(-x)$$

6. If the expression given defines a function, find its domain.

   (a) Mapping each student in the classroom to the seat in which they are sitting.
   Function; domain: the set of students in the classroom (assuming none are not sitting in a chair or are sitting in multiple chairs)

   (b) $f(x) = \frac{x^2+1}{x^2-4}$
   Function; domain: $\mathbb{R} - \{\pm 2\}$ or–equivalently–$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

   (c) $f(x) = \frac{x^{10}+x^4+x^3+x+11}{x-1}$
   Function; domain: $\mathbb{R} - \{1\}$ or–equivalently–$(-\infty, 1) \cup (1, \infty)$.

7. After years of intense research, UC-Berkeley's science faculty have determined that the 'awesomeness' of logic (L) is a linear function of the amount of time you've spent studying logic (S). In particular, scientists believe this function to be $L = \frac{8}{5}S + 10$.

   (a) Sketch a graph of this function

   (b) What is the slope of the graph and what does it represent?
   The slope of the graph is $\frac{8}{5}$ (note that the equation above is in slope-intercept form). This means there is an increase of $\frac{8}{5}$ in the awesomeness of logic for every 1 unit increase in time spent studying logic.

   (c) What is the S-intercept of the graph and what does it represent?

The S-intercept of the graph is found by replacing L with 0 and solving as below:

$$\begin{aligned}
0 &= \frac{8}{5}S + 10 \\
-10 &= \frac{8}{5}S \\
S &= \frac{-50}{8} \\
S &= -\frac{25}{4}
\end{aligned}$$

and represents the number of hours studying when the awesomeness of logic is 0.

8. Let $f(x) = \frac{x^2}{x-1}$ and define the domain of $f(x)$ as the real line ($\mathbb{R}$). Is $f(x)$ a function? Why or Why not?

   No. Because the domain was defined as the entire real line, the domain contains $x = 1$–a value for which f is not defined.

9. Let $f(x) = x^3 - 4$, $g(x) = x^2$. Find $f \circ g(x)$ and $g \circ f(x)$.

$$f \circ g(x) = f(g(x)) = (x^2)^3 - 4 = x^6 - 4$$

$$g \circ f(x) = g(f(x)) = (x^3 - 4)^2$$

10. Simplify the following:

    (a) $x^5(x^4) = x^{5+4} = x^9$

    (b) $\frac{x^{-2}}{x^{-4}} = \frac{x^4}{x^2} = x^{4-2} = x^2$

    (c) $\frac{4^{-3}}{2^{-6}} = \frac{2^6}{4^3} = \frac{2^6}{2^{2^3}} = \frac{2^6}{2^6} = 1$