Worksheet 16: Derivative Applications

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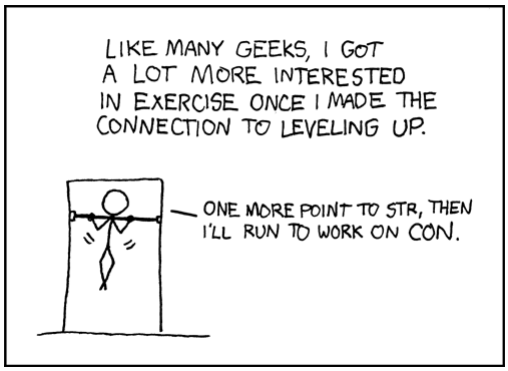

www.xkcd.com

1. Find the derivative by implicit differentiation: $x^3 + x^2y + 4y^2 = 6$.

2. Find the formula for the nth derivative $f^{(n)}(x)$ if $f(x) = \frac{1}{3x^3}$

3. Differentiate $f(x) = \ln(\ln(\ln(x)))$

4. A bacteria culture grows with constant relative growth rate. The bacteria count was 400 after 2 hours and 25,600 after 6 hours.
   (a) What is the relative growth rate?

   (b) What was the initial size of the culture?

   (c) Find an expression for the number of bacteria after t hours.

   (d) Find the rate of growth after 4.5 hours

   (e) When will the population reach 50,000?

5. Strontium-90 has a half-life of 28 days.

   (a) A sample has a mass of 50mg initially; find a formula for the mass remaining after t days.

   (b) How long does it take the sample to decay to a mass of 2mg?

6. (a) If A is the area of a circle with radius r and the circle expands as time passes, find $\frac{dA}{dt}$ in terms of $\frac{dr}{dt}$.

   (b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of $\frac{1m}{s}$, how fast is the area of the spill increasing when the radius is 30m?

7. Suppose $4x^2 + 9y^2 = 36$ where x and y are functions of t.

   (a) If $\frac{dy}{dt} = \frac{1}{3}$, find $\frac{dx}{dt}$ when $x = 2$ and $y = \frac{2}{3}\sqrt{5}$.

   (b) If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = -2$ and $y = \frac{2}{3}\sqrt{5}$.

8. Find the line tangent to the curve $f(x) = (1 + 3x)^{10}$ at $(0, 1)$.

9. ($\star$) Find the third degree polynomial Q such that $Q(1) = 1$, $Q'(1) = 1$, $Q''(1) = 6$, and $Q'''(1) = 12$.