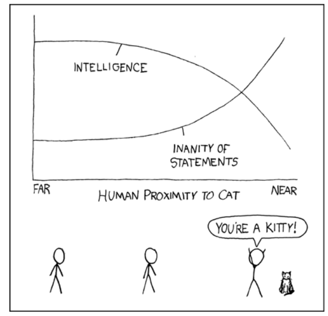
Worksheet 13: Trigonometric Derivatives & Chain Rule

Russell Buehler
b.r@berkeley.edu



www.xkcd.com

1. Find the derivative:

   (a) $y = 2\sec(x) - \csc(x)$

   (b) $f(\theta) = \sin(\theta)\cos(\theta)$

   (c) $f(\theta) = \sin(\theta)\csc(\theta)$

   (d) $y = \frac{1-\sec(x)}{\tan(x)}$

2. Evaluate: $\lim_{x \to 0} \dfrac{\sin 4x}{\sin 6x}$

3. Evaluate: $\lim_{x \to 0} \dfrac{\sin x^2}{x}$

4. Write the chain rule in both Leibniz and Newtonian notation.

5. Find the derivative:

   (a) $y = (x+1)^{10}$

   (b) $f(x) = e^{-x}$

(c) $y = (x^2 + e^{2x-1})^2$

(d) $f(\theta) = \sin(\cos(\tan(\theta)))$

(e) $y = 2^x$

(f) $f(x) = ((3x^5 + e^{2x} + x^4)^{12} + 2x)^3$

6. You've learned about composing two functions, e.g., if $f(x) = x^2$ and $g(x) = x + 1$, then $f \circ g(x) = (x+1)^2$. Decomposing a function reverses this process. For instance, if $h(x) = (x+1)^2$ then $f \circ g(x)$, where f and g are as above, is called a decomposition of h.

   (a) $h(x) = (x+1)^2$ can be decomposed in several other ways. Find one.

   (b) Can a function *always* be decomposed in more than one way? For instance, can $h(x) = x$ be decomposed in more than one way? Give an example of a function that can't be decomposed in two ways, or explain why you think that all functions can be decomposed in more than one way.

7. Before the midterm, you found the derivative of $f(x) = |x|$ by cases; find the derivative of $f(x)$ with the chain rule instead.

8. Using Leibniz notation, find the derivative of $x^2 + y^2 = 1$ without solving for y. Why is Leibniz notation good for implicit differentiation?