Worksheet 10: Basic Derivatives

Russell Buehler
b.r@berkeley.edu

www.xkcd.com

1. If you haven't already, write the general form for:
   (a) The Power Rule
   $\frac{d}{dx}x^r = rx^{r-1}$ for all real numbers r
   (b) The Constant Multiple Rule
   $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$ if $f(x)$ differentiable and c a constant
   (c) The Sum Rule
   $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ if $f(x)$, $g(x)$ differentiable.
   (d) The Difference Rule
   $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$ if $f(x)$, $g(x)$ differentiable.

2. Find the first and second derivative of: $f(x) = 3x^2$. Express them in both major notations.
   $f'(x) = \frac{d}{dx}f(x) = 6x$; $f''(x) = \frac{d^2}{dx^2}f(x) = 6$

3. Find the first and second derivative of: $f(t) = 2e^t - 5$. Express them in both major notations.
   $f'(t) = \frac{d}{dt}f(t) = 2e^t$; $f''(t) = \frac{d^2}{dt^2}f(t) = 2e^t$

4. Find derivative; express it in both major notations.
   (a) $f(x) = \frac{x^2+x+1}{x} = x + 1 + x^{-1}$
   $f'(x) = \frac{d}{dx}f(x) = 1 - x^{-2}$
   (b) $f(p) = 3p - \sqrt{p}$
   $f'(p) = \frac{d}{dp}f(p) = 3 - \frac{1}{2}p^{-\frac{1}{2}}$
   (c) $B(a) = 5e^a + \sqrt{a} + 6a^2$
   $B'(a) = \frac{d}{da}B(a) = 5e^a + \frac{1}{2}a^{-\frac{1}{2}} + 12a$

5. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y - 1 = 3x$

   Since the line $y - 1 = 3x$ has slope 3, we're looking for the tangent line with slope 3. To find this point, we can use the derivative (recall that the derivative gives the slope at x). Taking the derivative of y, we obtain $y' = \frac{3}{2}x^{\frac{1}{2}}$. Setting this equal to our desired slope:

$$\frac{3}{2}x^{\frac{1}{2}} = 3$$
$$x^{\frac{1}{2}} = 2$$
$$x = 4$$

   We may obtain the y-value corresponding to $x = 4$ by plugging into the original function, $y = 4\sqrt{4} = 8$. To get the equation of the tangent line we plug our point into $y = 3x + b$, obtaining

$$8 = 3(4) + b$$
$$-4 = b$$

   The final solution is thus $y = 3x - 4$.