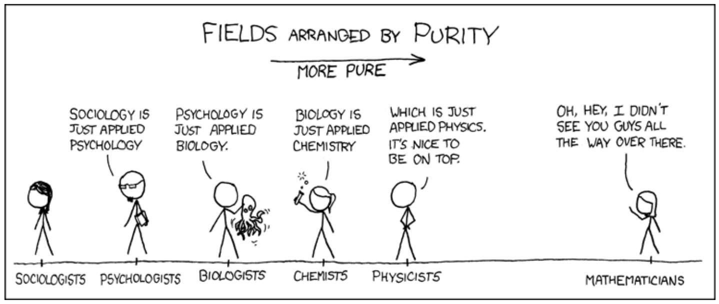
Worksheet 1: A Review of PreCalc

Russell Buehler

b.r@berkeley.edu



www.XKCD.com

1. Which of the following are functions?
   (a) $f(x) = x^3 + x + 1$

   (b) $f(x) = \begin{cases} 1 & : x \leq 1 \\ 5 & : 1 < x \leq 2 \\ 0 & : x \geq 2 \end{cases}$

   (c) $y = \frac{1}{5}x + \sqrt{7}$

   (d) $x^2 + y^2 = 1$

   (e) $x = f(x)^3$

   (f) $f(x) = cos(x)$

   (g) $x = f(x)^2$

   (h)

x	f(x)
1	1
2	1
3	2
4	3
5	5
6	8

   (i)

x	f(x)
5	1
2	1
6	2
7	3
3	5
42	8
2	2

   (j)

   (k)

   (l)

   (m)

   (a) Function. D: $\mathbb{R}$, R: $\mathbb{R}$

   (b) Not a function because $f(2) = 5$ and $f(2) = 0$.

   (c) Function. D: $\mathbb{R}$, R:$\mathbb{R}$

   (d) Not a function because, for instance, if $x = 0, y = \pm 1$

   (e) Function. D: $\mathbb{R}$, R: $\mathbb{R}$

   (f) Function. D: $\mathbb{R}$, R:$[-1, 1]$

   (g) Not a function because if $x = 4, f(x) = \pm 2$

   (h) Function. D: $\{1, 2, 3, 4, 5, 6\}$, R:$\{1, 2, 3, 5, 8\}$

   (i) Not a function because $f(2) = 2$ and $f(2) = 1$.

   (j) Not a function because if fails the vertical line test.

   (k) Function.

   (l) Function.

   (m) Not a function.

2. For each of the functions in 1, find the function's domain and range.

3. Over which parts of their domain are (e), (h), (k), and (l) increasing?

   (e) $\mathbb{R}$ (all of it)

   (h) $\{1, 2, 3, 4, 5, 6\}$ (all of it)

   (k)

   (l)

4. Let $f(x)$ be the line intersecting $(-2, 2)$ and $(1, 0)$. Find an algebraic expression for $f(x)$.

   Recall that the slope of a line is given by $\frac{(y_1-y_2)}{(x_1-x_2)}$ where (x_1, y_1) and (x_2, y_2) are points on the line. It follows that the slope of the line is $\frac{2-0}{-2-1} = -\frac{2}{3}$. Remembering that the slope of a line is m in the slope-intercept form $y = mx + b$, we now have $y = -\frac{2}{3}x + b$. Substituting the point $(1, 0)$ into the formula now allows us to solve for b:

$$0 = -\frac{2}{3}(1) + b$$

$$0 = -\frac{2}{3} + b$$

$$\frac{2}{3} = b$$

   And thus,

$$y = -\frac{2}{3}x + \frac{2}{3}$$

5. Sketch the graphs of $f(x) = x^3$, $f(x) = x^3 - 1$, and $f(x) = x^3 + 1$. Note how the graph changed.

6. Define $f(x) = x^2$. Sketch $f(x), f(2x)$, and $f(x + 4)$. Note how the graph changed.

7. Generalize your observations from 5 and 6 for $f(ax), f(x + a)$, and $f(x) + a$ where a is a real number.

   $f(ax)$ either stretches or shrinks the graph vertically depending on whether $a > 1$ or $0 > a < 1$. If $a < 0$, $f(ax)$ is mirrored across the y-axis and stretched or shrunk as before.

   $f(x + a)$ shifts the graph a units along the x-axis.

   $f(x) + a$ shifts the graph a units along the y-axis.