1. Have \(-\frac{1}{x}\sin(lnx)e^{\cos(lnx)}\) by chain rule.

2. Since \(y'(x) = \frac{1}{x^2}(1 - \ln x)\), slope of tangent line at 1 is 1. Therefore \(y = x - 1\).

3. Taking derivatives of both and solving for \(y'\) obtain \(y' = \frac{ye^{x/y} - y}{xe^{x/y} - y^2}\). Noting \(e^{x/y} = x - y\) this can be rewritten as \(y' = \frac{xy - 2y^2}{x^2 - xy - y^2}\).

4. Have \(y(t) = y(0)e^{-0.0005t} = e^{-0.0005t}\).

5. Set \(f(x) = \sqrt{x}\). Have \(f(x) \approx f(4) + f'(4)(x - 4)\) for \(x\) near 4. Therefore
\[
 f(4.1) \approx 2 - \frac{1}{4.10} = \frac{79}{40}. 
\]

6. \(\circlearrowleft\) Note \(f(-3) = 9\) and \(f(5) = 65\). \(\circlearrowleft\) Since \(f'(x) = 3x^2 - 12\), have \(f'(c) = 0\) if and only if \(c = \pm 2\). Note \(f(2) = -16\) and \(f(-2) = 16\). \(\circlearrowright\) Comparing the values conclude absolute max is 65 and absolute min is -16.

7. Set \(f(x) = e^x + x\). \(\circlearrowleft\) If \(f(x)\) had more than one root, then \(f(x)\) would have at least two. Since \(f(x)\) differentiable on all numbers, Rolle’s theorem implies there exists a point \(c\) where \(0 = f'(c) = e^c + 1\). However \(e^c + 1 > 1 > 0\) for all numbers \(x\). Conclude \(f(x)\) has at most one root. \(\circlearrowright\) Note \(f(1) > 0\) and \(f(-1) < 0\). Since \(f(x)\) continuous on all numbers, I.V.T implies that \(f(x)\) has a root.

8. By L’Hospital’s rule have
\[
\lim_{x \to 0} \frac{\tan x - x}{x^3} = \lim_{x \to 0} \frac{\sec^2 x - 1}{3x^2} = \lim_{x \to 0} \frac{2 \sec^2 x \tan x}{6x} = \lim_{x \to 0} \frac{4 \sec^2 x \tan^2 x + 2 \sec^4 x}{6} = 1/3. 
\]

9. Note \(y = \sqrt{x(x-1)(x+1)}\). Domain \(\mathbb{R}\). Zeros at \(x = 0, \pm 1\). \(\lim_{x \to +\infty} y(x) = +\infty\) and \(\lim_{x \to -\infty} y(x) = -\infty\). Since \(y'(x) = \frac{1}{3}(x^3 - x)^{-2/3}(3x^2 - 1)\), have critical points \(x = \).
0, ±1, ±\(\frac{1}{\sqrt{3}}\) with
\[ y' > 0 \text{ for } |x| > \frac{1}{\sqrt{3}} \quad \text{and} \quad y' < 0 \text{ for } |x| < \frac{1}{\sqrt{3}}. \]

Local max at \(-\frac{1}{\sqrt{3}}\) and local min at \(\frac{1}{\sqrt{3}}\). Vertical tangent at 0, ±1. Obtain graph plotted here:

http://www.graphsketch.com/?eqn1_color=1&eqn1_eqn=root(x^3-x)\text{3}&eqn2_color=2&eqn2_eqn=&eqn3_color=3&eqn3_eqn=&eqn4_color=4&eqn4_eqn=&eqn5_color=5&eqn5_eqn=&eqn6_color=6&eqn6_eqn=&x_min=-1.5&x_max=1.5&y_min=-1&y_max=1&x_tick=.5&y_tick=.5&x_label_freq=1&y_label_freq=1&do_grid=0&do_grid=1&bold_labeled_lines=0&bold_labeled_lines=1&line_width=4&image_w=850&image_h=525

10. Set \(g(x) = \frac{\ln x}{x}\). Domain \(x > 0\). Zero at \(x = 1\). \(\lim_{x\to+\infty} g(x) = 0\) and \(\lim_{x\to0^+} g(x) = -\infty\). Since \(g'(x) = \frac{1}{x^2}(1 - \ln x)\) have critical point at \(x = e\) with \(g'(x) > 0\) for \(x < e\) and \(g'(x) < 0\) for \(x > 3\). Note local max at \(x = e\). Obtain graph plotted here:

http://www.graphsketch.com/?eqn1_color=1&eqn1_eqn=ln(x)%2Fx&eqn2_color=2&eqn2_eqn=&eqn3_color=3&eqn3_eqn=&eqn4_color=4&eqn4_eqn=&eqn5_color=5&eqn5_eqn=&eqn6_color=6&eqn6_eqn=&x_min=0&x_max=4&y_min=-10&y_max=10&x_tick=1&y_tick=1&x_label_freq=1&y_label_freq=1&do_grid=0&do_grid=1&bold_labeled_lines=0&bold_labeled_lines=1&line_width=4&image_w=850&image_h=525

Set \(f(x) = e^{g(x)}\). Domain \(x > 0\). Never zero. \(\lim_{x\to+\infty} f(x) = 1\) and \(\lim_{x\to0^+} f(x) = 0\). Note \(f'(x) = g'(x)f(x)\) by chain rule where \(f(x) > 0\) for all numbers \(x\). Have critical point at \(x = e\) with \(f'(x) > 0\) for \(x < e\) and \(f'(x) < 0\) for \(x > 3\). Local max at \(x = e\). Obtain graph plotted here:

http://www.graphsketch.com/?eqn1_color=1&eqn1_eqn=exp(ln(x)%2Fx)&eqn2_color=2&eqn2_eqn=&eqn3_color=3&eqn3_eqn=&eqn4_color=4&eqn4_eqn=&eqn5_color=5&eqn5_eqn=&eqn6_color=6&eqn6_eqn=&x_min=0&x_max=40&y_min=0&y_max=5&x_tick=2&y_tick=5&x_label_freq=2&y_label_freq=1&do_grid=0&do_grid=1&bold_labeled_lines=0&bold_labeled_lines=1&line_width=4&image_w=850&image_h=525