1. The graph of \( f \) has a horizontal tangent precisely when \( f'(x) = 0 \). Since \( f'(x) = 1 - 2 \cos(x) \), this happens when \( 1 - 2 \cos(x) = 0 \), i.e. \( \cos(x) = 1/2 \). The values of \( x \) which satisfy this are \( x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \), for \( n \in \mathbb{Z} \).

2. We have \( y' = 10(1 + 3x)^3(3) \) by the Chain Rule, so \( y'(0) = 10(1 + 3 \cdot 0)^3(3) = 30 \). The equation of the tangent line is \( y - 1 = 30(x - 0) \), or \( y = 30x + 1 \).

3. Taking derivatives implicitly yields
\[
\frac{1}{2\sqrt{xy}} \cdot \frac{d}{dx} [xy] = \frac{d}{dx} [x^2y] \\
\frac{1}{2\sqrt{xy}} (y + x \frac{dy}{dx}) = 2xy + x^2 \frac{dy}{dx} \\
y + x \frac{dy}{dx} = 2\sqrt{xy}(2xy + x^2 \frac{dy}{dx}) \\
(x - 2x^2 \sqrt{xy}) \frac{dy}{dx} = 4(xy)^{3/2} - y \\
\frac{dy}{dx} = \frac{4(xy)^{3/2} - y}{x - 2x^2 \sqrt{xy}}
\]

4. Notice \( y = \frac{x}{2x - 1} = \frac{1}{2} \left( \frac{2x}{2x - 1} \right) = \frac{1}{2} \left( \frac{2x - 1 + 1}{2x - 1} \right) = \frac{1}{2} \left( 1 + \frac{1}{2x - 1} \right) \). Therefore \( y' = -(2x - 1)^{-2}, y'' = 4(2x - 1)^{-3}, y''' = -24(2x - 1)^{-4} \).

5. By the Chain Rule,
\[
\frac{d}{dx} [\ln(\ln(\ln(x)))] = \frac{1}{\ln(\ln(x))} \frac{d}{dx} [\ln(\ln(x))] \\
= \frac{1}{\ln(\ln(x))} \frac{1}{\ln(x)} \frac{d}{dx} [\ln(x)] \\
= \frac{1}{\ln(\ln(x)) \cdot \ln(x)} \frac{1}{\ln(x)} \frac{d}{dx} [\ln(x)] \\
= \frac{1}{x \cdot \ln x \cdot \ln(\ln(x)) \cdot \ln(\ln(x))}
\]

6. By the Product Rule, \( \frac{d}{dx} [\sinh(x) \tanh(x)] = \frac{d}{dx} [\sinh(x)] \tanh(x) + \sinh(x) \frac{d}{dx} [\tanh(x)] = \cosh(x) \tanh(x) + \sinh(x) \text{sech}^2(x) = \sinh(x)(1 + \text{sech}^2(x)) \).

7. Let \( f(x) = \sqrt{x} \), and \( a = 100 \). Then \( f(a) = 10 \), and \( f'(a) = \frac{1}{2\sqrt{100}} = \frac{1}{20} \), so the linear approximation to \( f \) at \( a \) is \( L(x) = f(a) + f'(a)(x - a) = 10 + \frac{1}{20}(x - 100) \). Since \( 99.8 \approx 100 \), \( \sqrt{99.8} = f(99.8) \approx L(99.8) = 10 + \frac{1}{20}(99.8 - 100) = 10 + \frac{1}{20}(-0.2) = 10 - 0.01 = 9.99 \).

Alternative approach (with differentials): For \( f(x) \) as above, we have \( dy = f'(x)dx = \frac{dx}{2\sqrt{x}} \).

For \( a = 100, x = 99.8 \), we have \( dx = \Delta x = -0.2 \), so \( \sqrt{99.8} = \sqrt{100} + \Delta y \approx 10 + dy = 10 + \frac{-0.2}{2\sqrt{100}} = 9.99 \).
8). We first find the critical numbers of \( f \). Since \( f'(x) = 3x^2 - 3 \), \( f'(x) = 0 \) when \( x = 1 \) or \( x = -1 \). As we only consider values in \([0, 3]\), the only critical number we check is \( x = 1 \). Evaluating at the critical number and the endpoints, we find \( f(0) = 1 \), \( f(1) = -1 \), \( f(3) = 19 \), so the absolute minimum is \(-1\) and the absolute maximum is \(19\).

9). We compute \( f'(x) = \frac{1}{3x^{2/3}} - \frac{2}{3x^{1/3}} \). \( f' \) is undefined for \( x = 0 \), and is 0 when \( \frac{1}{3x^{2/3}} = \frac{2}{3x^{1/3}} \) iff \( x^{1/3} = 2x^{2/3} \) if \( x = 8x^2 \) if \( x = 0, 1/8 \). The critical numbers are thus \( 0, 1/8 \).

10). As \( f \) is a polynomial, it is continuous on \([0, 4]\) and differentiable on \((0, 4)\). Also \( f(0) = 1 = f(4) \), so \( f \) satisfies the hypotheses of Rolle's Theorem on \([0, 4]\). The conclusion is then that there exists at least one value \( c \) in \((0, 4)\) with \( f'(c) = 0 \). We have \( f'(x) = 2x - 4 \), which is 0 precisely when \( c = 2 \).

11). \( f'(x) = 2xe^x + e^x x^2 = xe^x(2 + x) \), so \( f' = 0 \) when \( x = 0 \), \(-2 \). We see that \( f'(x) < 0 \) for \(-2 < x < 0 \) (e.g., substitute \( x = -1 \)), and \( f'(x) > 0 \) when \( x > 0 \) or \( x < -2 \). So by the First Derivative Test, \((-2, 4/e^2)\) is a local maximum and \((0, 0)\) is a local minimum for \( f \), and \( f \) is increasing on \((-\infty, -2) \cup (0, \infty)\) and decreasing on \((-2, 0)\).

12). Since \( e^x - 1 - x \big|_{x=0} = 0 = x^2 \big|_{x=0} \), we may use L'Hospital's Rule (0/0 indeterminate form) to evaluate the limit. We have \( \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{e^x - 1}{2x} \) if the latter limit exists. Again, since \( e^x - 1 \big|_{x=0} = 0 = 2x \big|_{x=0} \), we apply L'Hospital's Rule again to conclude that \( \lim_{x \to 0} \frac{e^x - 1}{2x} = \lim_{x \to 0} \frac{e^x}{2} = \frac{1}{2} \). Thus the original limit is \( \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2} \).

13). Notice \( \sin(x), \sinh(x) \) are continuous functions on \( \mathbb{R} \), and \( \sin(0) = 0 = \sinh(0) \). Thus we substitute \( x = 0 \) to obtain \( \lim_{x \to 0} \frac{\sin(x)}{\sinh(x) + 1} = 0 + 1 = 0 \).

14). Domain: \( f \) is undefined when \( 1 + \cos(x) = 0 \), which occurs when \( x = (2n + 1)\pi \), for \( n \in \mathbb{Z} \). Thus the domain of \( f \) is \( \{x \in \mathbb{R} \mid x \neq (2n + 1)\pi, n \in \mathbb{Z}\} \).

Local Extrema: \( f'(x) = \frac{(1 + \cos(x))(\cos(x)) - \sin(x)(-\sin(x))}{(1 + \cos(x))^2} = \frac{\cos(x) + 1}{(1 + \cos(x))^2} \).

This is always \( > 0 \), and is undefined when \( 1 + \cos(x) = 0 \), precisely where \( f \) is undefined. Thus \( f \) is always increasing, and has no local maxima or minima.

Behavior at infinity: Both \( \cos(x), \sin(x) \) are periodic of period \( 2\pi \), so \( f \) is also periodic with the same period. Also, \( \sin(x) \) is odd and \( 1 + \cos(x) \) is even, so \( f \) is odd. Thus the graph of \( f \) is just obtained by horizontal translates of its restriction to \([-\pi, \pi]\). \( f \) also has vertical asymptotes at \( x = (2n + 1)\pi \), \( n \in \mathbb{Z} \).

Zeros: \( f \) has zeros where it is defined and \( \sin(x) = 0 \), i.e. when \( x = 2n\pi \), \( n \in \mathbb{Z} \).

Behavior at 0: As seen above, \( f \) has a root at 0, and is continuous at \( 0 \) (and increasing in a neighborhood of \( 0 \)).

15). Domain: \( \{x \in \mathbb{R} \mid x > 0\} \) (we only look at \( x > 0 \))

Local Extrema: \( f(x) = e^{\ln(x)} \Rightarrow f'(x) = e^{\ln(x)} \left(1 - \ln(x)\right) = \frac{x^{1/2}(1-\ln(x))}{x^2} \). Thus \( f' = 0 \) when \( x = e \). For \( 0 < x < e \), \( f'(x) > 0 \), and for \( x > e \), \( f'(x) < 0 \). Thus \( f \) has a local max at \((e, e^{1/e})\), is increasing on \((0, e)\), and decreasing on \((e, \infty)\).

Zeros: \( f(x) = e^{\ln(x)} \) is never 0 for \( x > 0 \).

Behavior at \( \infty \): \( \lim_{x \to \infty} \frac{\ln(x)}{x} = 0 \), so \( \lim_{x \to \infty} f(x) = \lim_{x \to \infty} e^{\ln(x)} = e^0 = 1 \).

Behavior at 0: Substituting 0 for \( x \) gives the non-indeterminate form \( 0^\infty = 0 \), so \( f(0) = 0 \).