Name: _____

Student ID Number:_____

Instructions:

- 1. You will have 50 minutes to complete the exam.
- 2. The exam is a total of 4 questions and each question is worth 10 points.
- 3. Unless stated otherwise, you may use results we proved in class and on the homework.
- 4. No books, notes, calculators, or electronic devices are permitted.
- 5. If you require additional space, please use the reverse side of the pages.
- 6. The exam has a total of 6 pages with the last page reserved for scratch work. Please verify that your copy has all 6 pages.

Question	Score	Points
1.		10
2.		10
3.		10
4.		10
Total		40

1. (a) Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a function. For $r \in \mathbb{N}$, state what it means for f to be a C^r -diffeomorphism. (b) Fix $n \in \mathbb{N}$ and let $\sigma \in S_n$ be a permutation on $\{1, 2, \ldots, n\}$. Show that $f: \mathbb{R}^n \to \mathbb{R}^n$ defined by

$$f(x_1,\ldots,x_n)=(x_{\sigma(1)},\ldots,x_{\sigma(n)})$$

is a C^r -diffeomorphism for every $r \in \mathbb{N}$.

2. (a) For $S \subset \mathbb{R}^2$, say what it means for S to be **Riemann measurable**. (b) Let $R = [0,1] \times [0,1] \subset \mathbb{R}^2$ and fix $a, b \in \mathbb{R}$. Define $f \colon \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) := \begin{cases} a & \text{if } x \leq y \\ b & \text{otherwise} \end{cases}.$$

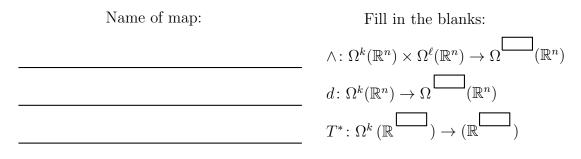
Show that f is Riemann integrable on R and compute $\int_R f$.

- 3. (a) For $k, n \in \mathbb{N}$, give the definition of a *k*-cell in \mathbb{R}^n .
 - (b) Let $\varphi \in C_2(\mathbb{R}^2)$ be defined by

 $\varphi(u_1, u_2) = (u_1 \cos(2\pi u_2), u_1 \sin(2\pi u_2)).$

Let $\omega = f dy_{(1,2)} \in \Omega^2(\mathbb{R}^2)$ with $f(y_1, y_2) = y_2$. Compute $\int_{\varphi} \omega$.

4. (a) Let $k, \ell, n, m \in \mathbb{N}$ and let $T \colon \mathbb{R}^n \to \mathbb{R}^m$ be a smooth map. Give the name of each map and fill in the blanks:



(b) For

$$\omega = f_1 dy_{(1,2)} + f_2 dy_{(1,3)} + f_3 dy_{(2,3)} \in \Omega^2(\mathbb{R}^3),$$

with

 $f_1(y_1, y_2, y_3) = y_1^2 y_3$ $f_2(y_1, y_2, y_3) = y_2 y_3$ $f_3(y_1, y_2, y_3) = y_1 y_3$

compute the ascending presentation of $d\omega$.