

249 Replacement Week 10 Problems

April 4, 2016

1. Show that a uniform matroid is transversal.
2. Let \mathcal{A} be a family of subsets of a set S . Let $\Delta[\mathcal{A}]$ be the bipartite graph associated to \mathcal{A} and $M[\mathcal{A}]$ the associated transversal matroid. Characterize the circuits of $M[\mathcal{A}]$ in terms $\Delta[\mathcal{A}]$.
3. Let M be a matroid and $\omega : E(M) \rightarrow \mathbb{R}$ a one-to-one function. Prove that M has a unique basis of maximum weight.
4. Let $\mathcal{A} = (A_j \mid j \in J)$ be a family of subsets of a set S . Show that $S - A_j$ is a flat of $M[\mathcal{A}]$ for all $j \in J$.
5. Show that if F and F' are flats of a matroid M , then so is $F \cap F'$.
6. (hard) Let B be a maximum-weight basis of a matroid M and H be a hyperplane of M . Let X be the set of maximum-weight elements of $E(M) - H$. Prove that $X \cap B$ is nonempty.