

## 249 Replacement Week 7 Problems

March 14, 2016

Problems from Stanley, EC Volume I.

3.108, 3.109, 3.110, 3.114, 3.115

Other problems (taken from previous courses by L. Williams and M. Haiman).

1. Let  $\mathcal{A}$  be an arrangement and  $\text{ess}(\mathcal{A})$  its essentialization. Show that

$$t^{\dim(\text{ess}(\mathcal{A}))} \chi_{\mathcal{A}}(t) = t^{\dim(\mathcal{A})} \chi_{\text{ess}(\mathcal{A})}(t)$$

2. Let  $\mathcal{A}$  be an arrangement in  $\mathbb{R}^n$  with defining polynomial  $Q_{\mathcal{A}} = x_1 x_2 \cdots x_n$ . Compute the characteristic polynomial  $\chi_{\mathcal{A}}(t)$  and compute the number  $r(\mathcal{A})$  of regions of  $\mathcal{A}$ .
3. Let  $\mathcal{A}$  be an arrangement in  $\mathbb{R}^n$  with equations

$$x_1 = x_2, x_2 = x_3, \dots, x_{n-1} = x_n, x_n = x_1$$

Compute the characteristic polynomial  $\chi_{\mathcal{A}}(t)$  and compute the number  $r(\mathcal{A})$  of regions of  $\mathcal{A}$ .

4. Let  $\mathcal{A}$  be an arrangement in  $\mathbb{R}^n$  with  $m$  hyperplanes. Find the maximum possible number  $f(n, m)$  of regions of  $\mathcal{A}$ .
5. Let  $\mathcal{A}$  be an arrangement in the  $n$ -dimensional vector space  $V$  whose normals span a subspace  $W$ , and let  $\mathcal{B}$  be another arrangement in  $V$  whose normals span a subspace  $Y$ . Suppose that  $W \cap Y = \{0\}$ . Show that

$$\chi_{\mathcal{A} \cup \mathcal{B}}(t) = t^{-n} \chi_{\mathcal{A}}(t) \chi_{\mathcal{B}}(t)$$

6. Let  $\mathcal{A}$  be an arrangement in a vector space  $V$ . Suppose that  $\chi_{\mathcal{A}}(t)$  is divisible by  $t^k$  but not  $t^{k+1}$ . Show that  $\text{rank}(\mathcal{A}) = n - k$ .