

## 249 Replacement Week 6 Problems

February 29, 2016 (LEAP DAYYYYYY)

Problems from Stanley, EC Volume I.

3.183abde, 3.185adcfg

Other problems:

1. Show that the Bruhat order on the symmetric group is thin, i.e. every rank 2 interval is a diamond (equivalently every rank 2 interval has precisely 4 elements).
2. Prove that  $u < v$  iff  $u^{-1} < v^{-1}$  for  $u, v \in S_n$ .
3. (a) Show that the (strong) Bruhat order is *Eulerian* in the sense that its Möbius function is given by

$$\mu(\sigma, \omega) = (-1)^{\ell(\omega) - \ell(\sigma)}.$$

- (b) Calculate the Euler characteristic for the poset of  $S_n$  with the Bruhat ordering (see 3.8.6 Stanley).
4. Show that the number of  $r$ -planes in  $\mathbb{C}^m$  meeting each of  $r(m-r)$  general subspaces of dimension  $m-r$  nontrivially is

$$\frac{(r(m-r))!(r-1)!(r-2)! \cdots 1}{(m-1)!(m-2)! \cdots (m-r)!}$$

Hint: Use repeated Pieri's rule and the hook length formula.