249 Replacement Week 5 Problems

February 22, 2016

Problems from Stanley, EC Volume I.

3.76, 3.85, 3.87abc, 3.88

Other problems (taken from previous courses by L. Williams and M. Haiman).

1. Recall that in a poset P we say y *covers* x (written $y \gtrdot x$) if $y > x$ and there is no $z \in P$ with $y > z > x$. Let P be a locally finite poset. Define $\eta \in I(P)$ (the incidence algebra) by $\eta(x, y) = 1$ if y covers x and $\eta(x, y) = 0$ otherwise. Show that $(1-\eta)^{-1}(x, y)$ is equal to the total number of maximal chains in $[x, y]$.

2. Consider Young's lattice (the lattice of all partitions, ordered by containment). Calculate its Mobius function. That is, for each pair of partitions, $\lambda \subset \nu$, calculate $\mu(\lambda, \nu)$.