We use the identities $r^2 = x^2 + y^2$, $\sin \theta = y/r$ and $\cos \theta = x/r$. The equation $r = a \sin \theta + b \cos \theta$ becomes

\[
\begin{align*}
& \quad r = ay/r + bx/r \\
& \quad r^2 = ay + bx \\
& \quad x^2 - bx + b^2/4 + y^2 - ay + a^2/4 = a^2/4 + b^2/4 \\
& \quad (x - b/2)^2 + (y - a/2)^2 = \frac{a^2 + b^2}{4}.
\end{align*}
\]

This is the equation for a circle with center $(b/2, a/2)$ and radius $\sqrt{a^2 + b^2}/2$.

We first need to find the points where the curve intersects itself. Since the graph repeats itself after a rotation of $2\pi$, the points of intersection must occur at $r = 0$. Setting $r = \frac{1}{2} + \cos \theta = 0$ we see that the curve goes through 0 at $\theta = 2\pi/3$ and $\theta = 4\pi/3$. Thus the area inside large loop (including the area in the small loop) will be

\[
\frac{1}{2} \int_{-2\pi}^{2\pi} \left( \frac{1}{2} + \cos \theta \right)^2 d\theta = \frac{1}{2} \left( \frac{3\theta}{4} + \sin \theta + \frac{\sin 2\theta}{4} \right) \bigg|_{-2\pi}^{2\pi} = \frac{3\sqrt{3}}{8} + \frac{\pi}{2}
\]

while the area in the small loop will be

\[
\frac{1}{2} \int_{-2\pi}^{4\pi} \left( \frac{1}{2} + \cos \theta \right)^2 d\theta = \frac{1}{2} \left( \frac{3\theta}{4} + \sin \theta + \frac{\sin 2\theta}{4} \right) \bigg|_{-2\pi}^{2\pi} = -\frac{3\sqrt{3}}{8} + \frac{\pi}{4}.
\]

If we take the difference we get the area inside the large loop and outside the little one, $\frac{\pi}{4} + 3\sqrt{3}/4$. 

\[\text{Polar plot:}
\]