Problem 16.6 58

The surface breaks into 4 equal pieces, so we need only find the area of 1. Let us find the area of the cylinder parameterized by \( \vec{s}(u, v) = \langle u, \cos v, \sin v \rangle \) along the \( x \)-axis. The intersection of the cylinders \( x^2 + z^2 = 1 \) and \( y^2 + z^2 = 1 \) is on the planes \( x = \pm y \). This means that the domain of our parameterization has boundaries \( u = \pm \cos v \). The partial derivatives of \( \vec{s} \) are \( \vec{s}_u = \langle 1, 0, 0 \rangle \) and \( \vec{s}_v = \langle 0, -\sin v, \cos v \rangle \), so \( dS = |\vec{s}_u \times \vec{s}_v| \) \( du dv = dudv \). The area of the surface is

\[
\int \int_S 1dS = 4 \int_{\pi/2}^{\pi/2} \int_{-\cos v}^{\cos v} dudv = 16.
\]

Problem 16.8 18

From the parameterization of \( C \) we have \( z(t) = \sin 2t = 2 \sin t \cos t = 2x(t)y(t) \) for all \( t \), so \( C \) lies on the surface \( z = 2xy \). If we let \( S \) be the graph of \( z = 2xy \) over the unit disk then \( \partial S = C \). We can parameterize this surface by \( \vec{s}(u, v) = \langle u, v, 2uv \rangle \) with domain \( D = \{(u, v): u^2 + v^2 \leq 1 \} \). The normal vector is \( d\vec{n} = \langle 1, 0, 2v \rangle \times \langle 0, 1, 2u \rangle \) \( du dv = \langle -2v, -2u, 1 \rangle \) \( du dv \). By drawing a picture and the right hand rule we see that the correct orientation of the normal vector has vertical component pointing downwards, so we take \( d\vec{n} = \langle 2v, 2u, -1 \rangle \) \( du dv \). By Stoke’s theorem we have

\[
\int_C (y + \sin x, z^2 + \cos y, x^3) \, d\vec{r} = \int \int_S \text{curl}((y + \sin x, z^2 + \cos y, x^3)) \cdot \langle 2v, 2u, -1 \rangle \, du dv
\]

\[
= \int \int_D \langle -2z, -3x^2, -1 \rangle \cdot \langle 2v, 2u, -1 \rangle \, du dv
\]

\[
= \int \int_D \langle -4uv, -3u^2, -1 \rangle \cdot \langle 2v, 2u, -1 \rangle \, du dv
\]

\[
= \int \int_D -8uv^2 - 6u^3 + 1 \, du dv
\]

\[
= \int \int_D (-6r^4 \cos \theta - 2r^4 \cos \theta \sin \theta + 1) \, drd\theta = \pi
\]

Problem 16.9 32

\[
\vec{F} = -\int \int_S \rho g z \, d\vec{n} = -\int \int_E \langle 0, 0, \rho g \rangle \, dV = -\rho g \hat{k} \int \int_E 1 \, dV = -V \rho g \hat{k} = -W \hat{k}.
\]