Worksheet 8, Math 1B
Series Representations of Functions

Friday, March 9, 2012

1. (Reprinted from WS 7) Find the sums of the following series using differentiation, integration, and summation:

(a) \( \sum_{n=1}^{\infty} nx^{n-1}, \ |x| < 1 \)

(b) \( \sum_{n=1}^{\infty} nx^n, \ |x| < 1 \)

(c) \( \sum_{n=1}^{\infty} \frac{n}{2^n} \)

(d) \( \sum_{n=2}^{\infty} n(n-1)x^n, \ |x| < 1 \)

(e) \( \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} \)

(f) \( \sum_{n=1}^{\infty} \frac{n^2}{2^n} \)

2. Uses series to evaluate the limit

\[ \lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x} \]

3. Show that the function defined by

\[ f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \]

is not equal to its Maclaurin series.

4. Find the sums of the following series by comparing to known series:

(a) \( \sum_{n=0}^{\infty} \frac{3^n}{5^n n!} \)

(b) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{6^{2n} (2n)!} \)

(c) \( 1 - \ln x + \frac{(\ln x)^2}{2!} - \frac{(\ln x)^3}{3!} + \cdots \)