1. Find the radius of convergence and interval of convergence of the series:

   (a) \[ \sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}} \]
   (b) \[ \sum_{n=1}^{\infty} \frac{x^n}{5^n n^5} \]
   (c) \[ \sum_{n=1}^{\infty} \frac{n(x+1)^n}{4^n} \]

2. Suppose that the radius of convergence of the power series \( \sum c_n x^n \) is \( R \). What is the radius of convergence of the power series \( \sum c_n x^{2n} \)?

3. Find a power series representation for the function and determine the radius of convergence:

   (a) \( f(x) = \frac{1+x}{1-x} \)
   (b) \( f(x) = \ln(5-x) \)
   (c) \( f(x) = \frac{x^3}{(x-2)^2} \)

4. Find the sums of the following series:

   (a) \[ \sum_{n=1}^{\infty} nx^{n-1}, \ |x| < 1 \]
   (b) \[ \sum_{n=1}^{\infty} nx^n, \ |x| < 1 \]
   (c) \[ \sum_{n=1}^{\infty} \frac{n}{2^n} \]
   (d) \[ \sum_{n=2}^{\infty} n(n-1)x^n, \ |x| < 1 \]
   (e) \[ \sum_{n=2}^{\infty} \frac{n(n-1)}{2^n} \]
   (f) \[ \sum_{n=1}^{\infty} \frac{n^2}{3^n} \]

5. Fix an integer \( k > 0 \), and let \( f(x) = \sum_{n=0}^{\infty} c_n x^n \), where \( c_{n+k} = c_n \) for all \( n \geq 0 \). Assume that \( f \) is not a constant function. Find the interval of convergence of the series, and a formula for \( f(x) \).