Math 274: Tropical Geometry

UC Berkeley, Spring 2009
Homework # 2, due Tuesday, February 3

1. Let Σ be the 2-skeleton of the boundary of the 5-dimensional cube. Determine the integral homology of Σ. How about the homotopy type?

2. Let $K = \overline{\mathbb{Q}(t)}$. The following ideal in $K[x,y]$ describes the intersection of a "moving circle" and a "moving hyperbola" in the plane:
$$I \quad = \quad \big\langle\, (x-3t)^2 + (y-7/t)^2 - t\,,\; xy - t^3 \,\big\rangle.$$
Compute the variety $V(I)$. Represent each point by Puiseux series.

3. Can you find 3 triangles in $\mathbb{R}^3$ whose Minkowski sum has 27 vertices?

4. Determine the polyhedral subdivision $\Sigma(F)$ of $\mathbb{R}^3$ given by the tropical polynomial $F(w_1, w_2, w_3) \;=\; (w_1 \oplus w_2) \odot (w_1 \oplus w_3) \odot (w_2 \oplus w_3) \;\oplus\; 1$.

5. Consider the linear subvariety $\mathbb{Q}^5$ which is the row space of the matrix
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}$$
and let I be its vanishing ideal in $\mathbb{Q}[x_1, x_2, x_3, x_4, x_5]$. Compute an explicit Gröbner complex for I, and compute all the initial ideals $\mathrm{in}_w(I)$ associated to the vertices w of your polyhedral complex in $\mathbb{TP}^4$.

6. Consider the curve defined by the following ideal in $\mathbb{Q}[x^{-1}, y^{-1}, z^{-1}]$:
$$I \quad = \quad \big\langle \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\,,\; x^2 + y^2 + z^2 - 1 \big\rangle.$$
Find the ideals of its closures in $\mathbb{A}^3$ and $\mathbb{P}^3$. Find one point (a,b,c) on that curve where a,b,c are 2-adic numbers whose valuation is non-zero.

7. Download the `Singular` package `tropical.lib` due to Thomas Markwig, and illustrate its features by computing some non-trivial examples.