Math 55, **Second Midterm Exam**
Thursday, April 9, 8:10am–9:30am

This exam is closed book. You may not use any books, notes or electronic devices. Please write your answers in a blue note book. Write your name, the name of your TA and your section time on the cover. There are five problems, each worth 20 *points, for a total of* 100 *points. Answers without justification will not receive credit. Your graded exam will be returned in your discussion section on Monday, April 13.*

(1) Consider all permutations of the letters $ABCDEFG$.
   (a) How many of these permutations contain the strings ABC and DE (each as a consecutive substring)?
   (b) In how many permutations does A precede B (not necessarily immediately)?

(2) Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped. Find the expected value and the variance of X_n.

(3) Prove that $\log_2(5) = 2.3219...$ is not a rational number.

(4) Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with the initial conditions $a_0 = 1$ and $a_1 = 9$.

(5) A thumb tack is tossed until it first lands with its point down, at which time no more tosses are made. On each tack toss, the probability of landing point down in 1/3.
   (a) Find the probability that exactly five tosses are made.
   (b) What is the expected number of tosses?