Math 55, **First Midterm Exam**
Thursday, February 23, 8:10am–9:30am

This exam is closed book. You may not use any books, notes or electronic devices. Please write your answers in a blue note book. Begin by writing your name, the name of your TA and your section time on the cover. There are five problems, each worth 20 points, for a total of 100 points. Answers without justification will not receive credit. You may look at your graded exam in your discussion section on Wednesday, February 29.

(1) Express the negations of each of these statements so that all negation symbols immediately precede predicates:
  (a) $\forall x \exists y \forall z \, T(x, y, z)$
  (b) $\forall x \exists y \, P(x, y) \ \vee \ \forall x \exists y \, Q(x, y)$
  (c) $\forall x \ \exists y \, \big(P(x, y) \wedge \exists z \, R(x, y, z) \big)$
  (d) $\forall x \exists y \, \big(P(x, y) \ \rightarrow \ Q(x, y) \big)$

(2) Determine an integer n such that

$$n \equiv 1 \pmod 7, \quad n \equiv 3 \pmod 8 \quad \text{and} \quad n \equiv 2 \pmod 9.$$

(3) Which amounts of postage can be formed using only 5-cent and 6-cent stamps? Formulate a conjecture and prove it.

(4) Compute the following remainders:
  (a) 19^{145} **mod** 13
  (b) $(-12)^{36} \cdot 50^{19}$ **mod** 7

(5) Give an example of two uncountable sets A and B such that the intersection $A \cap B$ is
  (a) finite,
  (b) countably infinite,
  (c) uncountable.