Math 55, **Final Exam**
Wednesday, May 20, 8:00am–11:00am

This exam is closed book. Do not use any books, notes or electronic devices. Please write in a blue note book, with your name, the name of your TA and your section time on the front. Each of the 10 *problems is worth* 10 *points, for a total of* 100 *points. Answers without justification will receive no credit.*

(1) (a) How many nonisomorphic *unrooted* trees are there with 4 vertices ?
(b) How many nonisomorphic *rooted* trees are there with 4 vertices ?

(2) How many ways are there to distribute five balls into three boxes if each box must have at least one ball in it and
(a) the balls are labeled but the boxes are unlabeled?
(b) the balls are unlabeled but the boxes are labeled?
(c) both the balls and the boxes are unlabled?

(3) Give a recursive definition of the set of bit strings that are palindromes.

(4) Consider the *divisibility* poset $\big(\{1, 2, 3, 12, 18, 36\}, \mid\big)$.
(a) Draw the Hasse diagram of this poset.
(b) Determine whether this poset is a lattice.

(5) Determine the number of paths in the xy plane starting at the origin $(0, 0)$ and ending at the point (m, n). Here, each path is made up of a series of steps, where each step

is a move one unit to the right or a move one unit upward.

(6) A new employee checks the hats of *six* people at the opera, forgetting to put claim check numbers on them. When people come back for their hats, the checker returns hats chosen at random from the remaining hats.
(a) What is the probability that no one receives his own hat?
(b) What is the expected number of hats that are returned correctly?

(7) Find an integer x such that

$$x \equiv 0 \,(\mathrm{mod}\, 2), \;\; x \equiv 3 \,(\mathrm{mod}\, 7) \;\text{ and }\; x \equiv 6 \,(\mathrm{mod}\, 11).$$

(8) Let G_1 and H_1 be two isomorphic graphs, and let G_2 and H_2 be two isomorphic graphs. Prove or disprove that the graphs $G_1 \cup G_2$ and $H_1 \cup H_2$ are isomorphic.

(9) There are three 6-sided dice on a table. Two are regular fair dice, and the third is a loaded die that comes up **6** half of the time. You grab a die at random, try it, and roll a **6**. What is the probability that you have the loaded die?

(10) How many relations are there on the set $A = \{1, 2, 3\}$ that
(a) are symmetric?
(b) are reflexive and symmetric?
(c) are neither reflexive nor irreflexive?